

**APPLICATION OF NON-LINEAR LEAST SQUARE
METHOD TO ESTIMATE THE MUSCLE DYNAMICS OF
THE ELBOW JOINT**

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Abstract: This paper presents an original use of the non-linear least-squares method applied to the muscle dynamics of the human body. The human body dynamics is very complex because of the number of degrees of freedom and of the number of muscles, moreover the behavior of muscles is non-linear and subject specific. A dynamic model of muscle, commonly used by the biomechanics community, which is presented, gives a relation between muscle force, activity, length and velocity. An application to the flexion/extension of the joint elbow using four muscles is then proposed. The dynamic parameters of those four muscles are estimated experimentally by the non-linear least square method. The activity (input of the dynamic model of the muscle) is measured using electromyography. The human arm dynamics is analyzed in a motion capture studio which acquisition of movements allows to compute the inverse kinematics and the inverse dynamics. Finally the muscle force is estimated (input of the dynamic model of the muscle).

Keywords: musculo-tendon dynamics, inverse dynamics, musculo-skeletal human model, electromyogram, motion capture

1. INTRODUCTION

The latest advances in neuroscience and in biomechanics have permitted medical researchers to enhance their understanding of muscles diseases and to develop solutions to help people suffering from handicap or Parkinson disease. Despite it is possible for the latest to have good diagnosis and to qualify the damages by using visual analysis. For example for the observation of the muscle stiffness and the trembling, there is no experimental quantification of the affected muscles and no record of the muscles dynamics, for there is no easy to use system able to quantify those parameters. It is mainly due to the nature of the muscles themselves and the complexity of the models that must be

used, the high number of muscles and degrees of freedom, but also to the differences between each subject such as size, mass, strength, capacity and muscle fibers repartition which are also related to the subject's history: sport, injuries, metabolism. Unlike usual biomechanics works on the human body where the model is only globally scaled to the subject (size and mass, or physiologically meaningless parameters) (Lloyd and Bessier, 2003), the proposed method focuses on identifying the subject specific muscle dynamic parameters. It is based on the use of the Hill-Stroeve muscle model (Stroeve, 1996), a motion capture studio and the inverse dynamics and inverse kinematics of the human body described in (Yamane *et al.*, 2005).

The second section presents the musculo-skeletal modelling of the elbow joint with four muscles and the Hill-Stroeve muscle model. The experimental set up is described in the

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third section. Finally the identification method using non-linear least square method based on the Newton-Gauss algorithm and experimental results of the identification of the muscle dynamics are the purpose of the fourth section.

2. MUSCULO-SKELETON DYNAMIC MODELLING OF THE ELBOW JOINT USING MUSCLE MODEL

This paper focuses on flexion/extension (F/E) of the elbow joint. It has been chosen rather than the knee joint because even if the generated forces are smaller, less muscles are involved, therefore the anatomy and the modelling are less complex. It consists first in a musculo-skeleton model of the joint, then in the dynamic modelling of each muscle.

2.1 Musculo-tendon modelling of the elbow joint

The human elbow joint has three rotational degrees of freedom that allow the hand to move in a wide operational space and to do pronation and supination movements. Flexion and extension is the rotation around z axis (Fig.1). In the following it is the only considered movements.

According to human anatomy, F/E movements of the elbow joint involve four muscles (Fig.2):

- Biceps, Brachialis and Brachioradialis for the flexion,
- Triceps for the extension.

A first approach was to simplify the model by considering the two main antagonist muscles: Biceps and Triceps, but the first measurements have shown that it was impossible to restrict the arm to such a model as it is not possible to avoid the contribution of a muscle physiologically involved in a movement. The four muscles must then be considered. The joint dynamics is described by (1).

$$J\ddot{q} = T = T_{BR} + T_{BB} + T_{Br} + T_{TB} + T_{ext} - B_l\dot{q} \quad (1)$$

where J is the inertia of the forearm and the hand, q is the elbow joint angle, \dot{q} and \ddot{q} its first and second derivatives, T is the joint torque, $T_i = F_{ti}ma_i$ is the torque due to the musculo-tendon complex where $i = BR$ for Brachioradialis, $i = Br$ for Brachialis, $i = BB$ for Biceps and $i = TB$ for Triceps, F_{ti} is the force applied by tendon i , ma_i is the moment arm of musculo-tendon i on the moving part, T_{ext} the external torque due to external forces and gravity,

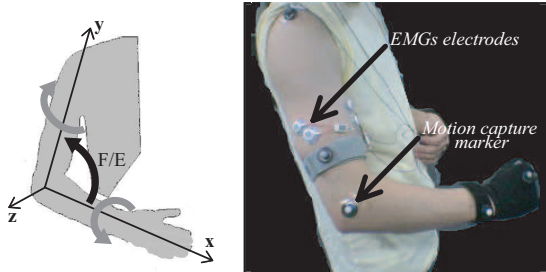


Fig. 1. The degrees of freedom of the elbow joint

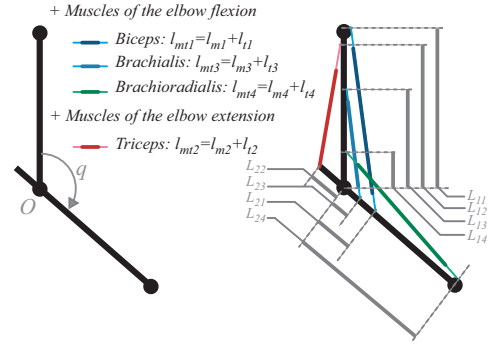


Fig. 2. The elbow joint with its 4 flexion and extension muscles

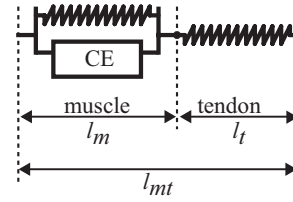


Fig. 3. Musculo-tendon complex without pinnation angle and B_l the viscosity of the joint. Moment arm ma_i and musculo-tendon complex length l_{mti} are computed using geometric considerations (2)(3).

$$ma_i = L_{1i} \frac{L_{2i}}{l_{mti}} \sin q \quad (2)$$

$$l_{mti} = \sqrt{L_{1i}^2 + L_{2i}^2 - 2L_{1i}L_{2i} \cos q} \quad (3)$$

2.2 Musculo-tendon complex dynamic modelling

Each muscle involved in the previous model (1) can be described by a musculo-tendon complex using the Hill-Stoeve model (Stroeve, 1996). This musculo-tendon complex is composed of a tendon and a muscle (Fig.3). The tendon is a passive wire that does not generate movement. The muscle is an active contractile element (CE) that generates contractions of the muscle controlled by neural excitation $u(t)$. According to the excitation and the desired movement the muscle lengthens ($\dot{l}_m > 0$) or shortens ($\dot{l}_m < 0$). Moreover, contractions of the muscle are assumed to be iso-volume (Zajac, 1989). From the neural excitation $u(t)$ the muscle activity $a(t)$ can be computed by a second order differential equation (Hirashima *et al.*, 2002). The musculo-tendon dynamics depends on muscle activity $a(t)$, and length and velocity of muscle and tendon, respectively l_m , \dot{l}_m , l_t and \dot{l}_t . The muscle model used here is a simplification of the Hill-Stroeve model: pinnation angle, the angle between the muscle and the tendon, is neglected. The force $F_m(t)$ developed by the muscle is function of muscle activity $a(t)$, the muscle length $l_m(t)$, the contraction velocity of the muscle $\dot{l}_m(t)$

and the maximal isometric force at full activation ($a(t) = 1$) F_{max} which is the maximal force that can be developed by the muscle for an isometric contraction ($\dot{l}_m = 0$).

$$F_m(t) = a(t)f_l(l_m)f_v(\dot{l}_m)F_{max} \quad (4)$$

with f_l is the force-length relation and f_v is the force-velocity relation given by:

$$f_l(l_m) = \exp\left(-\left(\frac{l_m - l_m^0}{l_m^{sh}}\right)^2\right) \quad (5)$$

$$f_v(\dot{l}_m) = \begin{cases} 0 & \text{if } \dot{l}_m \leq -v_{max} \\ \frac{V_{sh}(v_{max} + \dot{l}_m)}{V_{sh}v_{max} - \dot{l}_m} & \text{if } -v_{max} \leq \dot{l}_m \leq 0 \\ \frac{V_{sh}V_{shl}v_{max} + V_{ml}\dot{l}_m}{V_{sh}V_{shl}v_{max} + \dot{l}_m} & \text{if } \dot{l}_m \geq 0 \end{cases} \quad (6)$$

where v_{max} is the maximum contraction velocity, l_m^0 the optimal length of the muscle to be estimated, V_{sh} , V_{shl} , V_{ml} , and l_m^{sh} are the other subject specific dynamic parameters to be estimated.

Finally, considering the mass of the muscle M_m and its viscosity B_m , and applying the fundamental dynamic equation to the muscle, the following differential equation in the muscle length is obtained:

$$M_m \ddot{l}_m = F_t - F_m - B_m \dot{l}_m \quad (7)$$

According to Fig.3, length of the tendon can be computed by (8).

$$l_t = l_{mt} - l_m \quad (8)$$

Tendon force F_t is considered as lumped elasticity with stiffness k_t that is found in the literature (Zobitz *et al.*, 2001).

2.3 Simulation results

This model has been used to build a Matlab-Simulink simulator to study the musculo-tendon behavior and the possibility of estimating the dynamic parameters of the muscles using such modelling. It has also been used to design the exciting movements. Simulation data are given in Fig.4 for a flexion-extension movement of the elbow joint with alternate step neural input $u(t) = 0.5$ of flexors and extensor.

3. EXPERIMENTAL SETUP

The experimental set up has been designed in order to limit stress on subjects and to have easy to use system. It consists in some EMGs (ElectroMyoGraphy) electrodes to record the neural input $u(t)$ and some optical markers to record the joint position q with a motion capture system. The movements chosen for the identification are simple flexion-extension of the elbow similar to the one designed by simulation.

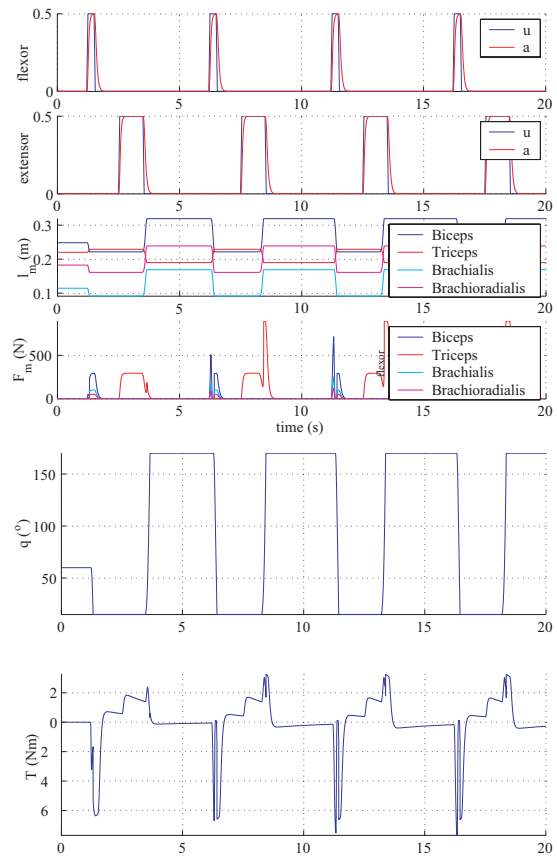


Fig. 4. Simulation of the flexion extension of the elbow joint. Top to bottom: Neural input $u(t)$ and muscle activity $a(t)$ for flexor, for extensor, muscle length l_m , muscle force F_m , joint angle q , joint torque T .

3.1 EMGs recording - Muscle activity

The muscle activation $u(t)$ is recorded using an EMGs system. This system uses surface electrodes that are fixed on the skin above the muscles (Fig.1). It can only give the neural input of the superficial muscles: Triceps, Biceps and Brachioradialis. A common approximation to estimate the neural input for deep muscles (Lloyd and Bessier, 2003) is to consider that Biceps and Brachialis are working together. Consequently they have the same neural input $u_{BB} = u_{Br}$ therefore the same activation $a_{BB} = a_{Br}$. To obtain good measurements the skin must be prepared: clean and scrub, and temperature and humidity of the room are better to be controlled. Despite those precautions there is some remaining noise. It can be efficiently removed using a low pass forward and reverse order 2 Butterworth, applied after the full wave rectification of the EMGs such as described in (Hirashima *et al.*, 2002) with 6 Hz cut off frequency.

3.2 Motion capture system - Joint angle, joint torque and muscle force

The in-house motion capture system is used (Yamane *et al.*, 2004). The whole system is capable of capturing the optical marker's position at 30 *fps* along with the data EMGs at 1 *KHz*.

Five optical markers, from the shoulder to the hand, are necessary (Fig.1) to record the elbow joint movements accurately. The inverse kinematics and the inverse dynamics models are computed by a musculo-skeletal model of the human body (Fig.5) using those measurements. With the optical markers position as input it gives the joint position q with the inverse kinematics and the joint torque T using the inverses dynamics. Finally \hat{F}_{ti} is the optimized solution for each tendon force computed from inverse dynamics data (Yamane *et al.*, 2005) and is the reference function for the identification (Equation (10) in Section 4.1).

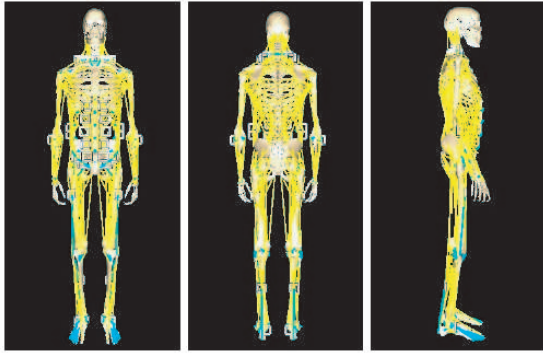


Fig. 5. Musculo-skeletal human model

3.3 Exciting movements for the estimation

The movements for the estimation are chosen in order to excite the dynamic parameters that have to be estimated using simulations. They also must verify the conditions that ensure the good estimation of the muscle forces such as:

- the tendon must have the smallest solicitation so that $F_{ti} \approx F_{mi}$,
- co-contraction of antagonist muscles must be limited to ensure that the optimal solution found by the optimization procedure is realistic (if co-contraction occurs infinite number of solutions is then found)

To ensure this latest condition one solution is to record movements that have been previously taught to the subject since it has been shown in (Osu *et al.*, 2002) that learned movements limit the co-contraction of antagonist muscles. Precise movements (grasping and depose tasks for example) also require a control pattern with co-contraction of the muscles. Such movements are avoided.

4. IDENTIFICATION OF MUSCULO-TENDON DYNAMICS

The parameters to be estimated are the subject specific constant parameters given in (5) and (6), that define the force-length and the force-velocity relations. Though, there are 5 parameters for each muscle to be estimated: \hat{l}_m^0 , \hat{l}_m^{sh} , \hat{V}_{sh} , \hat{V}_{shl} , \hat{V}_{ml} . F_{max} and v_{max} cannot be estimated, consequently values are from (Stroeve, 1996).

4.1 Identification method

Each muscle is considered separately (Brachioradialis, Triiceps, Biceps grouped with Brachialis). According to the expression of the muscle force given above in (6) the criteria must be separated in three cases. Cases when the muscle is not activated ($a_i = 0$) are excluded as they mean that the muscle is not activated therefore the muscle force is 0, and cases when $\dot{l}_m \leq -v_{max}$ are removed too, as they mean also that muscle force is 0. Thus, the two following cases remain:

$$f_v(\dot{l}_m) = \begin{cases} \text{case 1: } -v_{max} \leq \dot{l}_m \leq 0 : \frac{\hat{V}_{sh} (v_{max} + \dot{l}_m)}{\hat{V}_{sh} v_{max} - \dot{l}_m} \\ \text{case 2: } \dot{l}_m \geq 0 : \frac{\hat{V}_{sh} \hat{V}_{shl} v_{max} + \hat{V}_{ml} \dot{l}_m}{\hat{V}_{sh} \hat{V}_{shl} v_{max} + \dot{l}_m} \end{cases} \quad (9)$$

The system is sampled along a movement and leads to an over determinate system for each muscle. Several methods to solve this problem have been tested: simplex method (Venture *et al.*, 2005), simulated annealing and non-linear least square. For similar results the most efficient method is chosen: non-linear least square method implemented with a Newton-Gauss algorithm (Dennis, 1977) with the following criteria:

$$\min_{x \in \mathcal{R}^{n_p}} C(x) = \min_{x \in \mathcal{R}^{n_p}} \frac{1}{2} \sum_j (\hat{F}_t(t_j) - F_m(t_j, x))^2 \quad (10)$$

where j is the case $j = 1$ or $j = 2$, t_j is the sampled time for case j , \hat{F}_t is the tendon force estimated by the optimization routine of the inverse dynamic model of the human body and that is assumed to be the muscle force (section 3.3), F_m the muscle force given by (4) and x the vector of n_p parameters to be estimated. In case 1 $x = [\hat{l}_m^0 \hat{l}_m^{sh} \hat{V}_{sh}]$, in case 2 $x = [\hat{l}_m^0 \hat{l}_m^{sh} \hat{V}_{sh} \hat{V}_{shl} \hat{V}_{ml}]$.

The Newton-Gauss algorithm consists in minimizing the criteria C given by (10) by:

$$\min_{x \in \mathcal{R}^{n_p}} \|J(x)d - C(x)\|_2^2 \quad (11)$$

with J the Jacobian, d the direction of the search such as at computation step k $J(x_k)d_k = -C(x_k)$ and $x_{k+1} = x_k + d_k$.

Preliminary works have consisted in studying the identifiability of the parameters and the validity of the method with simulated data set generated by the Matlab-Simulink model (Section 2.3).

4.2 Experimental results

The muscle activity for the Triceps, the Biceps (assumed to be the same for the Brachialis) and the Brachioradialis is measured, rectified and filtered. The joint angle is computed using the inverse kinematics, the joint torque and the musculo-tendon forces are computed by the inverse dynamics. The movements for the estimation are chosen so that the flexor and the extensor muscles are working separately and are activated enough to develop a significant force that guaranty the identifiability of the parameters. They are achieved in the horizontal plan so that the effect of gravity are limited. The results presented here focus on exciting the muscles involved in the flexion only. Results are then given for the Triceps, the Brachialis (grouped with the Biceps) and the Brachioradialis. The maximal isometric force F_{max} is given in the literature for the upper limb (Stroeve, 1996) for a maximal activation. The initial values of vector x for each muscle are the values that can be found in (Stroeve, 1996). Results are given in tables 1 and 2 according to the case, for the Triceps, the Brachialis (grouped with the Biceps) and the Brachioradialis.

Table 1. Estimated parameters with C_1

muscle		\hat{l}_{m0i}	\hat{l}_{mshi}	\hat{V}_{shi}
Triceps	x_{init}	0.216	0.018	0.3
	\hat{x}	0.226	0.018	0.97
Brachialis	x_{init}	0.142	0.035	0.3
	\hat{x}	0.124	0.021	0.1
Brachioradialis	x_{init}	0.212	0.035	0.3
	\hat{x}	0.188	0.037	0.316

Table 2. Estimated parameters with C_2

muscle		\hat{l}_{m0i}	\hat{l}_{mshi}	\hat{V}_{shi}	\hat{V}_{shli}	\hat{V}_{mli}
Triceps	x_{init}	0.216	0.018	0.12		1.3
	\hat{x}	0.214	0.010	0.080		0.257
Brachialis	x_{init}	0.142	0.035	0.12		1.3
	\hat{x}	0.125	0.024	0.002		0.147
Brachioradialis	x_{init}	0.212	0.035	0.12		1.3
	\hat{x}	0.171	0.042	-1.29		1.014

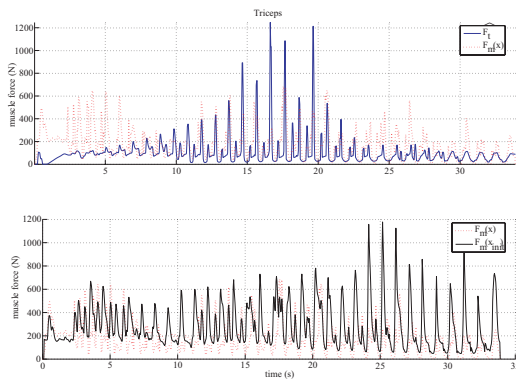


Fig. 6. Validation for the Triceps

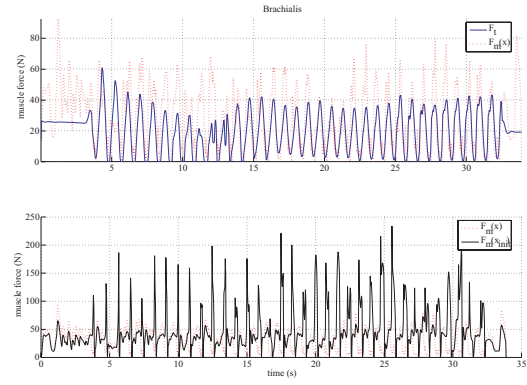


Fig. 7. Validation for the Brachialis

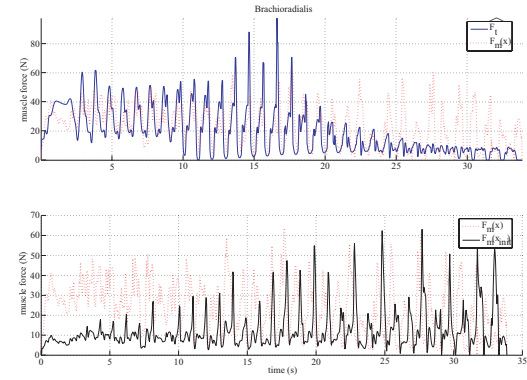


Fig. 8. Validation for the Brachioradialis

4.3 Interpretation and discussion

The obtained results show that the estimation is successful. The Newton-Gauss non-linear least square algorithm converges for each muscles with few computation steps and x smaller than 5×10^{-5} . Figures 6, 7 and 8 are validation of the results. The muscle force computed by (4) with the estimated parameters (red dashed line) and the one computed by optimization of the inverse dynamics (blue line (top)) are much more alike than the one computed by (4) with the parameters found in the literature (black solid line (bottom)). Parameters \hat{l}_{m0i} and \hat{l}_{mshi} are estimated in both cases of criteria. The same value for each case is found which shows the correctness of the model (ie. the value of v_{maxi}).

Despite, those results need improvements. The model is rather simple and does not take into account the full dynamics of the system: passive force, pinnation angle, variations optimal muscle length l_m^0 according to the level of activity (Lloyd and Bessier, 2003)... This step only treats the problem of muscle parameters, although the tendon slack length l_{t0} is very influent in the musculo-tendon dynamics. Conditions and assumptions of the tests: $F_{ti} \approx F_{mi}$ are restrictive and difficult to check and need to be simplified.

5. CONCLUSION

These preliminary experimental results have shown that with a simple model of musculo-tendon complex the identification of the subject specific parameters can be conducted. Oppositely to usual literature in this field the subject specific Hill-based model parameters are estimated. Method is based on the measurement of the muscle dynamics using EMGs data and motion capture system with simple tasks repeatable by everyone: flexion-extension of the elbow joint in the horizontal plan. Estimation is carried out for each muscle involved. The system obtained is a multi-variable non-linear over determinate system. It is solved by linear least square method with Newton-Gauss algorithm. Once those subject specific parameters are estimated, it will be possible to add the muscle model to the musculo-skeletal dynamic model used for the computation of the inverse kinematics and inverse dynamics and to improve the results of the computed muscle force. Such results allow to improve the knowledge of the human dynamics and of the muscle constraints for example during some specific movements. It is also very promising to apply such a method to people who have suffered injury to check their recovery, or to people with muscle disease. Further works concern improvements in the model to fit more easily with the subject and to be more physiologically correct. Therefore such results can be used by medical doctors studying sport science, rehabilitation, muscle diseases as they allow to understand, precisely simulate and control the muscle dynamics not from averaged data measured on a population of well chosen subjects but with subject specific parameters.

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