PARAMETRIC IDENTIFICATION OF THE CAR DYNAMICS

Gentiane Venture*   Maxime Gautier**
Wisama Khalil   Philippe Bodson***

**P.S.A. Peugeot Citroën – Direction Plates-formes, Techniques et Achats*
Route de Gisy – 78943 Vélizy-Villacoublay – France
venture@mpsa.com   bodson@mpsa.com

***Institut de recherche en Communications et Cybernétique de Nantes (IRCCyN)*
1, rue de la Noë – B.P. 92101 – 44321 Nantes Cedex 3 – France
Wisama.Khalil@irccyn.ec-nantes.fr   Maxime.Gautier@irccyn.ec-nantes.fr

Abstract: The aim of this paper is to give a general and unifying presentation of the identification issues of a passenger car using the Denavit and Hartenberg robotic formalism for the modelling. The desired parameters are the dynamic parameters of the car and the parameters of the suspension and the tires. The kinematics and dynamic models are automatically calculated using the software package SYMORO+. The identification is based on the use of the inverse dynamic model. Parameters are estimated using a weighted least squares method. The weighting procedure is developed. Practical results are given for a 406 Peugeot car.

Key words: parameter identification, automobile, dynamic modelling, least squares.

1. INTRODUCTION

The aim of this paper is to present a general and unifying methodology of modelling a car and to identify its dynamic parameters.
Several techniques of derivation of kinematics and dynamic models for mobile robots are available in the literature (Tilbury *et al.*, 1994, Zodiac, 1996), but the usual approach considers systems made up of a rigid cart and rigid wheels, moving in a horizontal plane, with the constraint of rolling without slipping. Meanwhile real working conditions of motion do not satisfy such hypothesis. Consequently, it is necessary that the dynamic model takes into account the 3D motion and the forces between the wheels and the soil and the aerodynamics.
With such a complexity, a systematic method of modelling, based on the modified Denavit & Hartenberg geometric description of a multi-body system (Khalil and Kleinfinger, 1986), facilitates the derivation of the dynamic and identification models. The car is considered as a tree structure made with 42 real or virtual bodies, where the four wheels are considered as the terminal links. This description allows to automatically calculate the symbolic expression of the geometric, kinematic and dynamic models by using robotics techniques or even by a symbolic software package like SYMORO+ (Symbolic Modelling of Robots) (Khalil and Creusot, 1997). The car suspensions, the anti-roll bars and the vertical behaviour of the tires are modelled with elasticities and with dampers if needed (Khalil and Gautier, 2000). Such a model allows to calculate the inverse dynamic model which is linear in the dynamic parameters, and to identify those parameters with a weighted least squares method, (Khosla, 1986, Khalil and Dombre, 2002, Guillo and Gautier, 2001).

2. DESCRIPTION OF A PASSENGER CAR USING MODIFIED DENAVIT & HARTENBERG NOTATIONS

The modified Denavit and Hartenberg (MDH) (Khalil and Kleinfinger 1986) notations can be applied to obtain the geometric parameters of a car. (Venture, and al. 2002).

Let R_0 be a fixed reference frame attached to the ground. The reference link C_r is the chassis, it corresponds to the body whose location ζ (i.e. position & orientation) gives the system posture in the frame R_0: $\zeta = [q_1\ q_2\ q_3\ q_4\ q_5\ q_6]^T$ for a movement in the 3D space. In that case the model of a car can be composed of a tree structure with 42 links C_j such that (Fig.1):

- C_0 is the base attached to the ground,
- $C_1, C_2, \ldots, C_5$ are virtual links used to define the car posture. Their variables $q_1\ q_2\ q_3\ q_4\ q_5$ correspond to the posture position variables $x\ y\ z$ and the angles $\theta\ \phi$, of the posture orientation variables respectively.
- C_6 is the chassis, its variable is the roll $q_6 = \psi$.
- C_9, C_{18}, C_{27} and C_{36} are the dampers, represented by linear springs and dampers.
- C_{14} and C_{23} are the rear wheels
- C_{32} and C_{41} are the front wheels

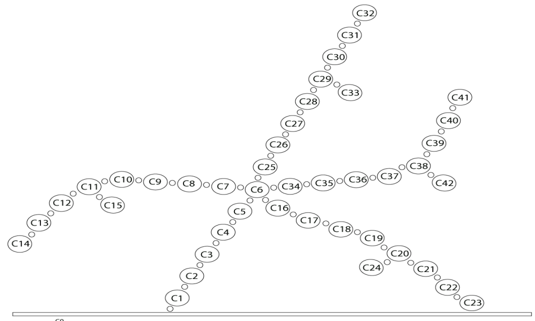

Figure 1: multi-bodies description of a car

- q_7, q_{16}, q_{25}, q_{34} are the ½ track width,
- q_8, q_{17}, q_{26}, q_{35} are the ½ wheelbase,
- q_9, q_{18}, q_{27}, q_{36} are the dampers clearances,
- q_{10}, q_{19}, q_{28}, q_{37} are the steering angles,
- q_{11}, q_{20}, q_{29}, q_{38} are the camber angles,
- q_{13}, q_{22}, q_{31}, q_{40} are the kingpin angles
- q_{14}, q_{23}, q_{32}, q_{41} are the rotation of the wheels
- q_{15}, q_{24}, q_{33}, q_{42} are the tire deflection.

This description allows calculating the geometric, kinematic and dynamic models automatically with the help of SYMORO+, software of symbolic calculations developed by the robotics team of the IRCCyN. (Khalil and Creusot, 1997)

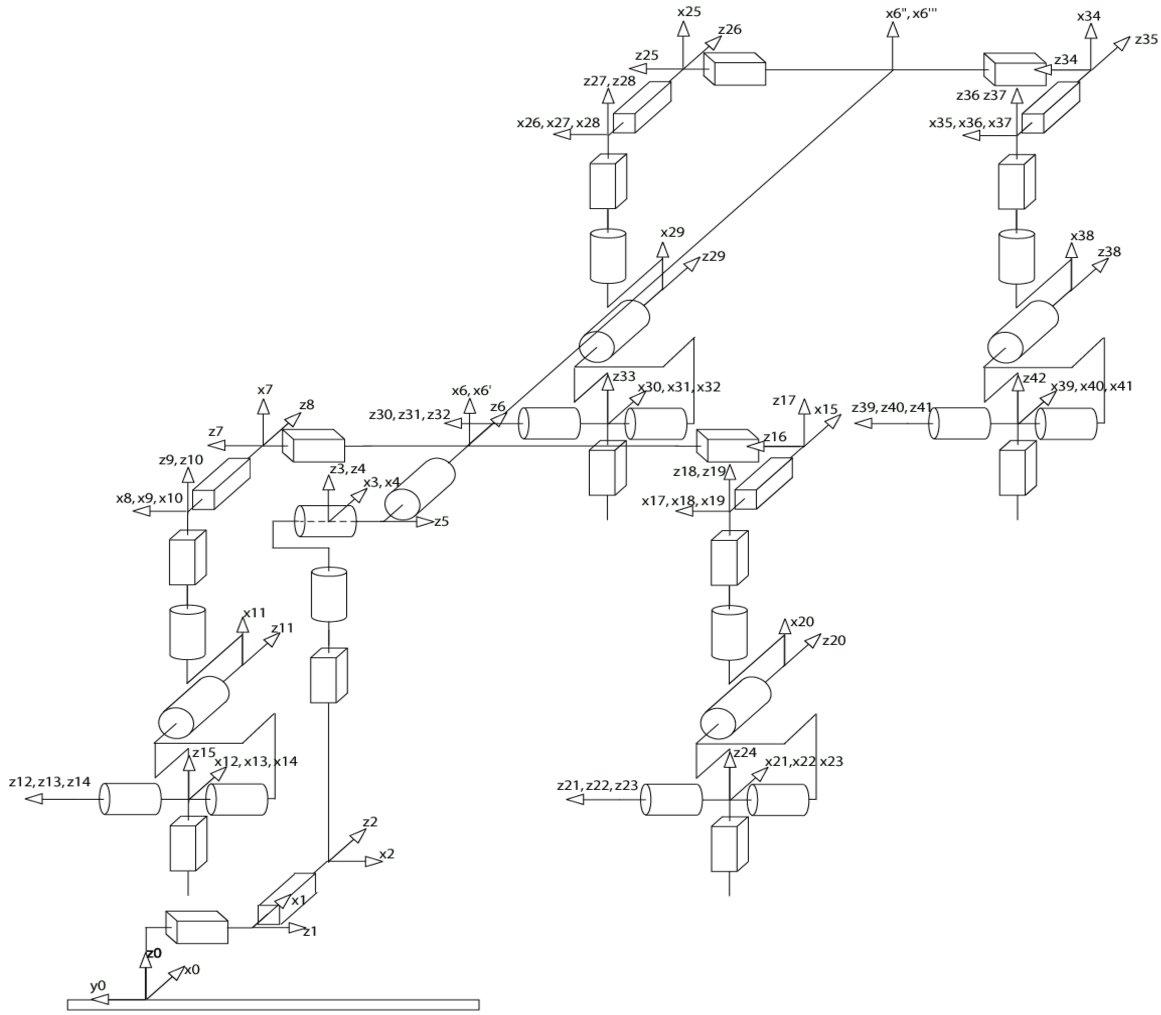

Figure 2: geometric description of a car

3. COMPUTATION OF THE DYNAMIC MODEL

The Inverse Dynamic Model (IDM) gives the joint torques as a function of the joint positions, velocities and accelerations. Let the IDM be written as:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{H}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{L} + \mathbf{Q}^c + \mathbf{Q}^a \quad (1)$$

- $\mathbf{M}(\mathbf{q})$ is the mass matrix of the system Σ
- $\mathbf{H}(\mathbf{q},\dot{\mathbf{q}})$ is the vector of centrifugal, Coriolis and gravity terms.
- **L** is the vector of the internal forces between the vehicle bodies: motor torque, friction, elasticity
- $\mathbf{Q^c}$ is the vector of the contact forces between the ground and the wheels projected on the joint axes.
- $\mathbf{Q^a}$ is the vector of aerodynamic forces projected on the joint axes.

3.1. Internal forces: L

The internal forces vector is composed of:

- the elastic forces

If j is an elastic joint, the j-component of $\boldsymbol{L^e}_j$, the elastic forces vector can be written:

$$\boldsymbol{L}^e_j = -k_j q_j - h_j qp_j$$

where q_j is the joint displacement w.r.t. the steady state position, k_j is the stiffness of the j-joint and h_j the damping coefficient.

- the friction vector component, is simplified as:

$$\boldsymbol{L}^f_j = -F_{vj}\dot{q}_j - F_{sj} sign(\dot{q}_j)$$

F_{vj} is the viscous friction coefficient.
F_{sj} is the Coulomb friction force.

3.2. Aerodynamic forces: $\mathbf{Q^a}$

The aerodynamic forces have to be taken into account for high-speed (> 100 km/h) tests. They are given by tables and applied on body C_6.

$$\mathbf{Q^a} = \mathbf{J_a}^T\mathbf{F^a}$$

where $\mathbf{F^a}$ is the aerodynamic forces and Ja is the Jacobian matrix corresponding to its projection on the joint axes.

3.3. Forces between the wheels and the ground: $\mathbf{Q^c}$

In the experiments, four dynamometric wheels measure the contact forces. They give the 6 directional forces and torques expressed in the frames R_8, R_{11}, R_{15}, R_{19}, attached to the wheel axes.

$$\mathbf{Q^c} = \Sigma \mathbf{J_{cj}}^T\mathbf{F^c_j}$$

where $\mathbf{F_j^c}$ is the contact forces on wheel j and $\mathbf{J_{cj}}$ is the Jacobian matrix corresponding to its projection on the joint axes.

3.4. Modelling of the anti-roll bars

The constraint equation (2) is simpler to add it by hand than to model it with the MDH notation.

$$F_{ar} = k_{ar}\,(q - q_{op}) \quad (2)$$

Where :

- F_{ar} is the anti-roll bar force,
- k_{ar} the anti-roll bar stiffness,
- q the damper clearance of the considered wheel
- q_{op} the damper clearance of the opposite wheel of the same axle.

It gives the complete equation for links 9, 18, 27, 36:

$$F_i = ks_i.q_i + h_i.qp_i + fs_i.\text{sign}(qp_i) + off_i + k_{ar}\,(q_i - q_{opi}) \quad (3)$$

4. DYNAMIC PARAMETERS

This is the parameters which are used to write the dynamic equations of the system.

4.1. Standard inertial parameters

For each link there are 10 standard inertial parameters (Gautier and Khalil, 1990), composed of :

- $[XX_j\ XY_j\ XZ_j\ YY_j\ YZ_j\ ZZ_j]$: the 6 elements of the inertia matrix of link j with respect to frame j,
- $[MX_j\ MY_j\ MZ_j]$: the 3 first moments of link j around the origin of frame j,
- M_j, the mass of link j.

Beside the inertial parameters, the dynamic parameters include for some joints the spring stiffness k_i, and dampers coefficients h_i, offset off_i, friction coefficients fs_i.

4.2. Base inertial parameters

The base inertial parameters are defined as the minimum inertial parameters that can be used to obtain the dynamic model. They represent the set of inertial parameters that can be identified using the dynamic model.

They are obtained from the standard inertial parameters by eliminating those that have no effect on the dynamic model and by grouping some others. There are two techniques to obtain those parameters: a symbolic one (Gautier and Khalil, 1990), or a numerical one (Gautier, 1991).

In the model concerned the base parameters are the standard parameters, there is neither regrouping relation nor elimination of parameters.

4.3. Base dynamic parameters

The base dynamic parameters, defining the vector of the dynamic parameters, X, to be identified, consists of:

- For the chassis (link C_6):
$\mathbf{X_6} = [XX_6\ XY_6\ XZ_6\ YY_6\ YZ_6\ ZZ_6\ MX_6\ MY_6\ MZ_6\ M_6]^T$
- For the wheels (links C_i, i = 14, 23, 32, 41):
$\mathbf{X_i} = [XX_i\ YY_i\ ZZ_i\ M_i]^T$
- For the suspensions (joints i = 9, 18, 27, 36):
$\mathbf{X_i} = [ks_i\ h_i\ fs_i\ off_i]^T$
- For the anti roll bars: $\mathbf{X_{ar}} = [k_{arf}\ k_{arr}]^T$
- For the tire deflection (joints i = 15, 24, 33, 42):
$\mathbf{X_i} = [kt_i]^T$

And finally X, the vector of identifiable parameters has the following expression:

$\mathbf{X} = [\mathbf{X_6}^T\ \mathbf{X_9}^T\ \mathbf{X_{14}}^T\ \mathbf{X_{15}}^T\ \mathbf{X_{18}}^T\ \mathbf{X_{23}}^T\ \mathbf{X_{24}}^T\ \mathbf{X_{27}}^T\ \mathbf{X_{32}}^T\ \mathbf{X_{33}}^T\ \mathbf{X_{36}}^T\ \mathbf{X_{41}}^T\ \mathbf{X_{42}}^T\ \mathbf{X_{ar}}^T]^T$

Let n_p be the number of the parameters to identify:

$$n_p = \text{length}(\mathbf{X}) \quad (4)$$

5. IDENTIFICATION

5.1. Identification model

The dynamic model (1) can be expressed as a linear relation w.r.t. the vector of identifiable parameters. It can be written as:

$$\mathbf{y} = \mathbf{D}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}).\mathbf{X} \tag{5}$$

5.2. Identification method : weighted least squares

X can be estimated, from the sampling of the dynamic model, as the least squares (L.S.) solution $\hat{\mathbf{X}}$, of the following linear system:

$$\mathbf{Y} = \mathbf{W}\mathbf{X} + \rho \tag{6}$$

- **W** is the observation matrix (n_{exp} x n_p), it represents the concatenation of the **D** matrix on the different samples of the trajectory.
- **Y** the (n_{exp} x 1) vector of joint and reaction forces
- ρ the (n_{exp} x 1) vector of model errors
- n_{exp} is the (number of samples*42)

The method allows computing a standard deviation for each parameter $\sigma_{\hat{X}i}$. Classically it is estimated considering **W** to be a deterministic matrix and ρ to be zero mean and independent noise, with standard deviation σ_p such that: $\mathbf{C_{pp}} = \sigma_p^2.\mathbf{I_{nexp}}$

The variance-covariance matrix can be calculated as:

$\mathbf{C}_{\hat{\mathbf{X}}} = \sigma_p^{\,2}.\left[\mathbf{W}^T.\mathbf{W}\right]^{-1}$,and $\sigma_{\hat{X}i}^{\;2} = C_{\hat{X}ii}$ is the i^th^ diagonal coefficient of $\mathbf{C}_{\hat{\mathbf{X}}}$. So the relative standard deviation $\sigma_{\hat{X}ri\%}$ is given by :

$$\sigma_{\hat{X}ri\%} = \frac{\sigma_{\hat{X}i}}{\hat{X}_i} \tag{7}$$

But, in fact, **Y** and **W** are obtained by concatenation of 42 equations that correspond of (j =1,42) linear systems with different standard deviation $\hat{\sigma}_p^j$. They can be calculated using the minimal 2-norm of errors calculated on each linear system:

$$\hat{\sigma}_p^j = \frac{\left\| Y^j - W^j.\hat{X}^j \right\|}{\sqrt{n_p^j - p^j}} \tag{8}$$

X^j is the (p^j x 1) vector of the minimal parameters of equation j

n_p^j is the number of equations of the linear system j

The inverse of these standard deviations give the weighting matrix **P**:

$$\mathbf{P} = \begin{bmatrix} S_1 & & \\ & \ddots & \\ & & S_{42} \end{bmatrix} \text{with } S_j = (1/\hat{\sigma}_p^j).I_{n_p^j} \tag{9}$$

Equation (6) weighted by P gives the following system:

$$\mathbf{Y_p} = \mathbf{W_p}\mathbf{X} + \rho_\mathbf{p} \tag{10}$$

where: $\mathbf{Y_p} = \mathbf{P}\,\mathbf{Y}$, $\mathbf{W_p} = \mathbf{P}\,\mathbf{W}$ *et* $\rho_\mathbf{p} = \mathbf{P}\,\rho$.

The solution $\hat{\mathbf{X}}_\mathbf{W}$ and the standard deviations $\hat{\sigma}_{\hat{X}Wi}$ are those given by the L.S. solution of (10). This weighting procedure allows improving both the estimation of the parameters and the estimation of the standard deviations.

6. EXPERIMENTAL SET UP

For the data acquisition, a real car is equipped with many sensors, which allows estimating the joint variables needed for the identification model and the contact forces with the ground.

6.1. Sensors and measurements

The sensors used in our case are composed of: four dynamometric wheels, an inertial unit, a laser sensor for vertical position, video camera "Zimmer", and a speed sensor "Correvit".

Table 1 : sensors characteristics

sensor	measurement	accuracy	Bandwidth	resolution
Inertial unit Sagem	Yaw speed *qp4*	0.01°/s	50°/s 10Hz	0.005°/s
	Roll speed *qp6*	0.01°/s	50°/s 10Hz	0.005°/s
	Pitch speed *qp5*	0.01°/s	50°/s 10Hz	0.005°/s
	Lateral acceleration rear axle *qdp1*	0.1m/s²	15m/s² 10Hz	0.01m/s²
	Longitudinal acceleration rear axle *qdp2*	0.02m/s²	15m/s² 10Hz	0.01m/s²
	Vertical acceleration rear axle *qdp3*	0.02m/s²	15m/s² 10Hz	0.01m/s²
	Absolute roll angle *q6*	0.1°	50° 5Hz	0.05°
	Absolute pitch angle *q5*	0.1°	50° 5Hz	0.05°
Speed sensor Correvit	Longitudinal speed *qp2*	1km/h	150km/h 5Hz	
	Lateral speed *qp1*		15m/s 5Hz	0.075m/s
Video camera Zimmer	Ground clearances *q9, q18, q27, q36*	1mm on smooth ground	125mm 200Hz	0.062mm
	Steering angles *q10, q19, q28, q37*	0.033°	5° 30 Hz	0.011°
	Camber angles *q11, q20, q29, q38*	0.042°	2.5° 30Hz	0.014°
Dynanometric wheel igel	Angular speed axles *q14, q23, q32, q41*		30tr/s 150Hz	
	Wheel total forces and torques	25N	20kN 150Hz	

Other joint coordinates are tabulated using the damper clearances and the steering angle. Tables of data are given by the car manufacturer (PSA) as a result of measurements carried out on a special test bench.

6.2. Filtering and joint coordinates estimation

All the measurements (position, velocity or acceleration) are filtered in order to estimate the position, the velocity and the acceleration for each joint. The derivatives are estimated with a zero-phase band pass filter whose transfer function is the product of a forward and reverse low pass butterworth filter (filtfilt matlab function) with a central difference algorithm. The order (between 5 and 8) and the cut off frequency (around 3Hz) are chosen according to the order of the derivatives and the dynamics to be identified (around 1Hz). The integrations are estimated with a trapezoidal method without phase shift. Once they are estimated it is possible to compute Y and W for the considered test.

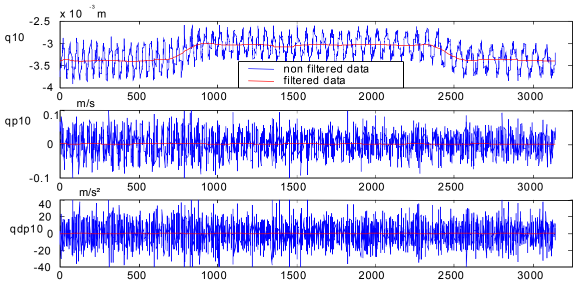

Figure 3 : filtering and derivation

7. RESULTS

7.1. Results

Results are obtained using standard tests such as sinus steering, straight line braking and spiral steering tests. That allow to limit the cost. The sample time is 0.0016 second, the number of samples: n_{exp}, depends on the tests. The estimated values and the relative standard deviations are shown in tables 2,3,4 and 5. Parameters are given in IS units and in the modelling frames.

Table 2: Inertial parameters of the chassis

parameter	*A priori*	estimated	%standard deviation
XX6	*6810.44*	6877.46	0.30
XZ6	*-445.9*	-590.92	0.56
YY6	*6567*	16697.76	1.72
ZZ6	*668.56*	714.26	1.01
MX6	*537.24*	357.43	0.29
MY6	*5.81*	-2.06	3.26
MZ6	*2368.21*	2446.61	0.07
M6	*1685*	1644.11	0.10

M6 is the mass of the whole car (spring mass and non-sprung mass). The position of the centre of mass in frame R6 is given as:

$X_{G/R6}$ = MX6/M6, $Y_{G/R6}$ = MY6/M6,
$Z_{G/R6}$ = MZ6/M6,

Table 3: dynamic parameters of the suspensions

parameter	*A priori*	estimated	%standard deviation
fs9		12.60	7.95
off9		589.08	0.38
ks9	*22000*	24425	1.42
h9	*4200*	4131.7	1.17
fs18		30.41	3.31
off18		-14.58	13.38
ks18	*22000*	20798	1.73
h18	*4200*	4445.2	1.13
karr	*19185*	21662	0.82
fs27		28.56	4.49
off27		420.94	1.67
ks27	*20000*	27478	1.32
h27	*3200*	3143.2	1.54
fs36		35.87	3.34
off36		1018.87	0.80
ks36	*20000*	26073	1.53
h36	*3200*	3869.1	1.36
karf	*19780*	18342	1.05

Table 5: vertical stiffness of the tires

parameter	*A priori*	estimated	%standard deviation
kt15	*200000*	216502.50	0.32
kt24	*200000*	217816.21	0.31
kt33	*200000*	232581.41	0.37
kt42	*200000*	230549.08	0.37

7.2. Interpretation and validation

A rule of thumb is to consider that the parameters whose standard relative deviations are greater than 20 times the smallest one are poorly identified. Most of all parameters are well identified with the given data. Another step of validation consists in comparing Y to the reconstructed vector $W.X$. (Figures 4,5,6,7,8).

Figures 4,5 and 6 give the results of a direct validation (carried out on the trajectories used in the identification). Figures 7 and 8 give the results of a cross validation, where the validation trajectory has not been used for the estimation: the test used for the cross validation is a braking in straight line while the identification test was a 90km/h sinus steering.

Abscises are $n_{exp} * n_{joint\ represented}$

Ordinates represent the joint force or the joint torque.

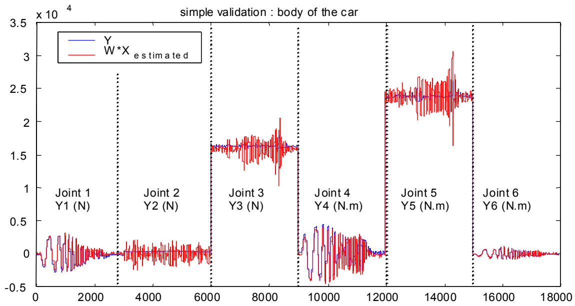

Figure 4: direct validation for the chassis

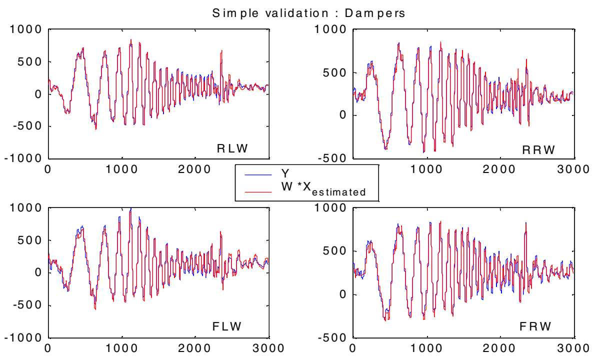

Figure 5: direct validation for the suspension

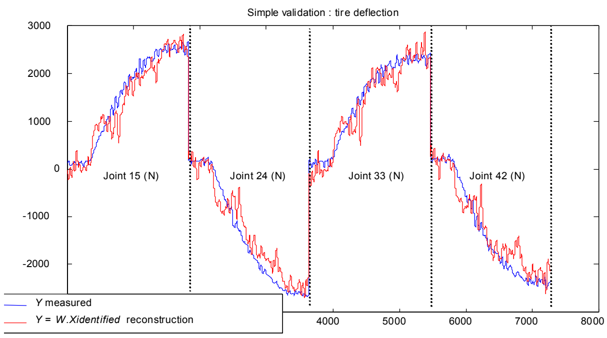

Figure 6: direct validation for the vertical stiffness of the tires

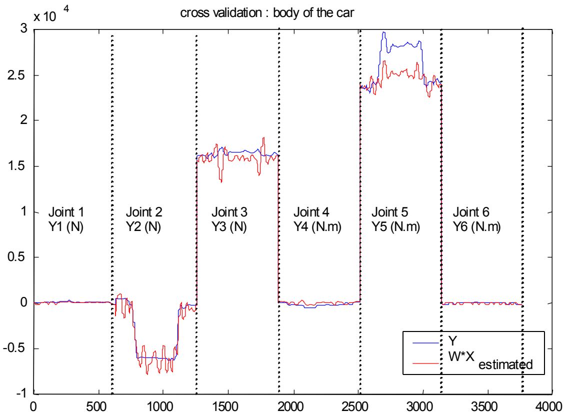

Figure 7: cross validation for the chassis

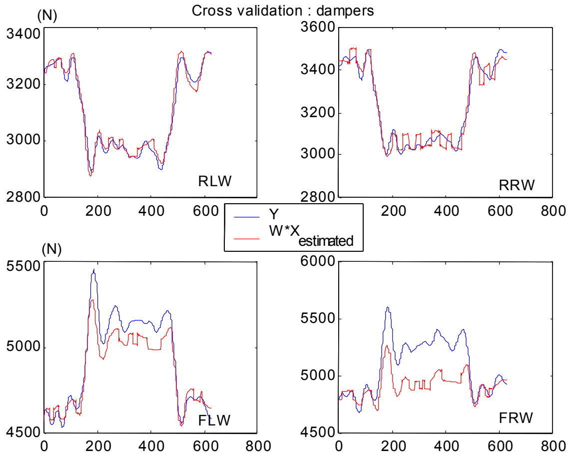

Figure 8: cross validation for the suspension

8. CONCLUSION

Results presented in this paper show the possibility of extending robotics formalism to the mobile robots and to the ground vehicles such as passenger cars. This formalism and the use of appropriated tools allow to compute automatically the dynamic models. The inverse dynamic model can be written as a linear relation w.r.t. the dynamic parameters, and the use of a weighted least squares technique is not time consuming and straight forward. Good results are obtained: they have small standard deviations, they are close to the a priori values, and they are confirmed by direct and cross validation.

The model has been validated and compared with respect to other dynamic models used for simulations.

Nevertheless, it is important to notice that the suspension stiffness and damping coefficients are supposed to be constant. This is a rough approximation particularly for the damping coefficients. It could be better to tabulate them from data obtained by special test measurements.

Further work is to extend the set of tires parameters in order to estimate tires characteristics.

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