Identification of Limbs Joint Passive Viscoelasticity: A Comparison of Two Experimental Methods

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Abstract

As the assessment of the human body limb dynamics is an important data and no unified method of quantification is yet used, in this paper we are comparing two methods to identify the human joint passive dynamics represented by a common linear model. The first method is a method broadly used in biomechanics or rehabilitation studies and is based on the use of a dynamometer during isokinetic movements. The second method is a in-house developed method based on the use of a motion capture system and a skeletal model to compute the joint angles and joint torques. In this paper we show that part of the success of both methods depends in the passivity of the movements (no activity of the muscles) and consequently in how much the subject can relax during the test. However with the first method it is difficult to guaranty such an hypothesis. We also show that in the measurements with the dynamometer the dynamics of the dynamometer is partially included which bias the obtained results and makes them difficult to use. We finally show that the proposed method is more reliable, easier to set up and to roll-out not only a medical environment but in a any kind of environment.

Introduction

The assessment of the limb passive viscoelasticity is a key data in medicine: in the evaluation of post-injury or post-stroke rehabilitation, in prosthesis and orthoses design and in the diagnosis and follow-up of neuromuscular diseases such as the cerebral palsy or the Parkinson disease. One of their symptoms is an increased muscle tone resulting in an increased viscoelasticity of the joints; during the clinical diagnosis the viscoelasticity of the joints is visually assessed by the neurologist using scales (Jones and Hunter, 1990), (Johnson, 2002). The most common is the Modified Ashworth scale but Scholtes et al. (Scholtes *et al.*, 2006) counted not less than 13 different scales in 119 articles which makes inter-rating and even intr-rating an issue, consequently quantification tools are highly required. The limb passive viscoelasticity is also important in the understanding of human motion and human gait as it is the elementary uncontrolled behavior underneath any active voluntary movement; and there too quantification is mandatory.

In biomechanics or in rehabilitation research the most conventional method to quantify the viscoelastic properties of the limb joints is to use a dynamometer or similar equipment (Such *et al.*, 1975), (Hayes and Hatze, 1977), (Xu and Hollerbach, 1998), (Lee *et al.*, 2002). These equipments offer the possibility to apply a torque on the joint and to measure the resulting displacement during isokinetic movements. Nevertheless they are often bulky machines that make inexperienced examinee feel uncomfortable; in addition only one joint at the time can be considered which makes experiments time consuming and tiresome for the examinee.

New techniques based on lighter and contact-less equipments such as optical motion capture systems (Venture *et al.*, 2006), or ultrasonic motion capture systems (Valle *et al.*, 2006), are very promising.

In this paper we propose to compare the results obtained with these two groups of estimation methods: the most common, based on the use of a dynamometer during passive isokinetic movements and a method using an optical motion capture system. The joint angle and joint torque in this latest are computed respectively by the inverse kinematics and the inverse dynamics using a skeletal model of the human body. The chosen movements are passive constraint-free movements. We apply these two method to the estimation of the right elbow passive dynamics. After giving the model used to describe the passive behavior of the joint, the two experimental systems and process are presented in details; finally obtained results with one experimented candidate are given and discussed.

Model of the joint and identification

Model of the passive viscoelasticity of limb joints

Though in biomechanics research a non-linear relationship is often used (Winters and Stark, 1985), (Stroeve, 1999) the viscoelasticity of the joints is represented here as a torsion spring-damper system such as in (Oatis, 1993). It is due to the combination of the elasticity, viscosity and Coulomb frictions of the passive musculo-tendon, the connective tissues and the soft tissues (Johns and Wright, 1962). They are respectively Γ^e, Γ^v and Γ^f such that for joint j,

$$\Gamma_j^e = k_j(q_j - q_j^r) = k_j q_j + o_j \tag{1}$$

with k_j the stiffness, q_j the joint angle, q_j^r the natural rest angle, $o_j = -k_j q_j^r$ the corresponding offset,

$$\Gamma_j^v = h_j \dot{q}_j \tag{2}$$

with h_j the viscosity and $\dot{q}_j$ the angular velocity of the joint.

$$\Gamma_j^f = f_j sign(\dot{q}_j) \tag{3}$$

with f_j the Coulomb coefficient of friction.

According to (Khalil and Kleinfinger, 1986) and (Khalil and Dombre, 2002), the identification model obtained from the inverse dynamics during passive movements is then:

$$Q - H = \Gamma^e + \Gamma^v + \Gamma^f \quad (4)$$

Where:

- Q is the vector of generalized efforts representing the projection of the external forces and torques on the joint axes. $Q = 0$ when there is no external effort applied.

- H is the vector of inertial, Coriolis, centrifugal and gravity forces, computed by scaling the candidate data with the Digital Human Center database.

Identification model

For the n_j joints the identification model given by Eq. (4) is linear in the n_p parameters to estimate: k_j, h_j, f_j, o_j for each concerned joint j. After sampling along a movement of n_e samples, we obtain a linear over-determinate system given by Eq. (5). The solution of this system $\hat{X}$ is found with the linear least square.

$$Y = W(q, \dot{q}, \ddot{q})X + \rho \quad (5)$$

where:

- Y is the $(n_e n_j \times 1)$ vector of joint torques, obtained by sampling $Q - H(q, \dot{q}, \ddot{q}, D_P)$,

- W is the $(n_e n_j \times n_j n_p)$ observation matrix (or regressor matrix), obtained by sampling Γ^e, Γ^v and Γ^f

- ρ is the $(n_e n_j \times 1)$ vector of modelling and measurements errors,

- X is the $(n_j n_p \times 1)$ vector of parameters to estimate, constituted of the n_j sets of joint parameters such as $X_j = [k_j\ h_j\ f_j\ o_j]^T$,

- $\hat{X}$ is the $(n_j n_p \times 1)$vector of estimates.

This formalism, commonly used in robotics, provides a limitation-free system in the number of concerned joints n_j and parameters to estimate n_p. The limitation depends only on the systems used for the measurements (the number of joint angles q and joint torques $Q - H$ that can be measured) and in the identifiability of the parameters according to the exciting properties of the movements chosen for the identification (Gautier and Khalil, 1992), (Swevers *et al.*, 1997).

The *condition number* of the observation matrix W and the *relative standard deviation* $\sigma_{\hat{X}_j}\%$ for each estimated parameter, ie for each component of vector $\hat{X}$, are computed and used to interpret the results (Venture *et al.*, 2006). Results are also interpreted using validation figures. They consist in comparing the vector of joint torques Y with the joint torque estimated from joint angle and identified joint dynamics: $W\hat{X}$. The resulting error $r = Y - W\hat{X}$ is also given. The error r takes into account measurements error and model error. In the case of direct validation the error is such that $r = \rho$ the least squares residual.

Experimental processes

EMG measurements

Surface EMG measurements record the neural activity of the muscle they are placed above. A passive movement is a movement of a joint without participation or effort on the part of the examinee thus without neural activity. Consequently the EMG records must show no significant activity of the muscles. To verify this condition we use a surface EMG system to measure the level of activity of the Biceps, the Brachioradialis and the Triceps during both experiments with the dynamometer and with the motion capture system. The record rate is $1\,KHz$. Muscle activity is obtained after full-wave-rectification and filtering (Hirashima *et al.*, 2002) of the recorded signal. The activity is then normalized by the maximal activity during maximal voluntary contraction that is also measured.

Dynamometer

The isokinetic torque-angular velocity relationship of the right elbow flexor/extensor muscles is measured by using an isokinetic dynamometer (Myoret, Kawasaki Heavy Industries, Japan). It includes a gravity compensation consequently the torque T that is measured includes the gravity term such that: $T = Q - H$ in the equation of dynamics Eq. 4.

With the help of an assistant, the examinee is positioned on the bench in order to make measurements of the right elbow: examinee lays on his back on the bench, his right upper arm rest on the bench. The axis of the elbow joint is visually aligned with the dynamometer axis, the examinee is then strapped on the bench, and the forearm is stripped on the lever of the dynamometer as shown in Fig. 1, left. The range of passive motion is determined by declaration of the examinee: $q_e \in [-5; 140]^\circ$. The range of motion chosen for the dynamometer is then $[-1; 121]^\circ$. A first sequence of active movements is performed to calibrate the dynamometer. The relationships between the measurements and the actual torque (flexion and extension are discriminated) and the joint angle are defined by Eq. (6). As the examinee is asked to perform maximal contraction of his muscles during the first sequence of active movements this movements are also used to measure the maximal voluntary contraction that serve to normalized the EMG data.

$$\begin{array}{llll} T_f = a_f T_m + b_f & \text{with} \quad a_f = -45.4 & \text{and} & b_f = -3.1 \\ T_e = a_e T_m + b_e & \text{with} \quad a_e = 52.1 & \text{and} & b_e = -9.3 \\ q = a' q_m + b' & \text{with} \quad a' = -18.0 & \text{and} & b' = 89.8 \end{array} \quad (6)$$

After a short rest to avoid fatigue phenomenon, the passive isokinetic movements are performed. They consist in a repetition of flexion/extension movements at $5^\circ/s$, $90^\circ/s$ and $180^\circ/s$ as shown in Fig. 1, right. The joint angle q, the angular velocity $\dot{q}$ and the joint torque T are measured.

Motion capture system and skeletal model

The optical motion capture system used for the experiments is a in-house system composed of 10 high-resolution cameras able to capture 30 fps but high speed cameras recording 200 fps can also be used. The examinee is equipped with a set of reflective optical markers of diameter 8 mm. For the

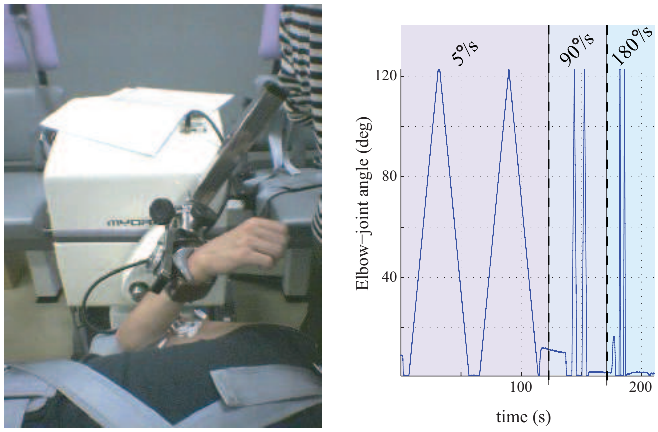

Figure 1: Left: The examinee stripped on the torque meter for right elbow measurements. Right: the joint angle profile of the passive isokinetic movements

recording of the right upper limbs 9 optical markers are positioned on the following anatomical points:

- the top of each the shoulders (on top of the acromion),
- each side (lateral and medial) of the elbows,
- each side of the wrists (ulna's radial side and radius radial side).
- the top of the hand (top of the capitate)
- on the top of each pelvis bone.

The markers on the hips (pelvis) and on the opposite shoulder (acromion) serve to define the appropriate trunk posture during the computation of the inverse kinematics.

After the labelling of the optical markers, we use a 155 degrees of freedom (dof) skeletal model of the human body (Fig. 2), to compute the inverse kinematics and the inverse dynamics according to (Nakamura and Yamane, 2000), (Yamane and Nakamura, 2003). We thus obtain the joint angle q and the corresponding joint torque H for each considered joint: in that case for the right wrist joint, the right elbow joint and the right shoulder joint. The angular velocity $\dot{q}$ of the joint is obtained by derivation of the joint angle q, using a centered difference algorithm. Boundary effects are removed from the data-set samples during $0.2\,s$ in each side.

By using the fact that the markers are positioned above fixed anatomical points we can scale the model to the actual subject. We use the distance between the optical markers to compute automatically the length of each segment and the marker file associated. It allows a considerable gain of time compared to manual scaling and manual description of the subject specific model and the marker file, and also to limit error. Results of the geometric scaling for the upper body are given in Table 1 and compared with direct measurements of the segments length. The relative difference D does not exceed 4% which makes the automatic scaling very reliable.

Constraint-free movements

To perform a good identification exciting movements must be use, however they must remain passive and painless. The

Table 1: Results of the geometric measurements with both automatic scaling from markers data and direct measurement of the length of segments of the upper body

Parameter	Unit	Scaled	Measured	D %
Height of the torso	m	0.463	0.456	2.9
Width of the torso	m	0.165	0.170	1.5
Length of upper arm	m	0.353	0.350	0.8
Length of lower arm	m	0.259	0.250	3.8

constraint-free movements are characterized by no external forces applied on the considered joints, thus $Q = 0$, such as shown Fig.3.

This experimental system allows the execution of any movement. As we focus on the estimation of the right elbow viscoelasticity during passive movements, we chose the free swing of the elbow as the constraint-free movement. The shoulder is maintained at the maximal extension without abduction, while the elbow is at rest position (about 90°). Before starting the test the forearm is lifted and maintained at the maximal extension by an assistant. When released, the forearm swings naturally around the rest position. These movements are an elbow joint equivalent of the knee joint pendulum tests (Wartenberg, 1951) often used in medicine or biomechanics.

Experiments

Candidate

The estimation is performed on a young healthy male (height 168cm, weight 58 Kg) who presents no symptoms of spasticity nor rigidity of the right arm. The examinee has already experienced measurements with both experimental set-ups. This is preferable for comparison of methods however this condition may not be maintained for roll-out. It allows to minimize the level of muscle contractions due to the stress of examination and in the case of the dynamometer, it also min-

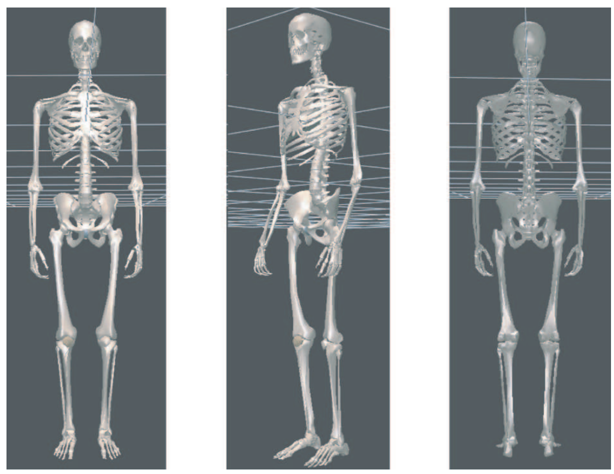

Figure 2: The skeletal model with 155 dof used to compute the human kinematics and dynamics from optical markers position

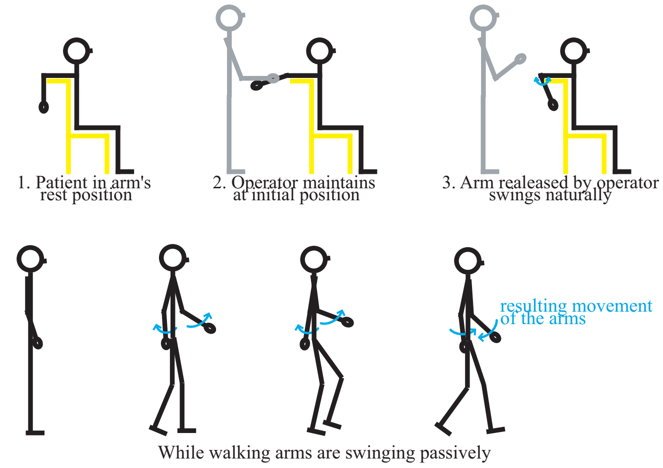

Figure 3: Examples of constraint-free movements of the arm: free swing of the elbow and free swing or the arm

imize the muscle contractions due to reflex, consequently it should guaranty the achievement of passive movements. Prior to the tests the examinee received detailed explanations of the experimental procedure.

Data post-processing

The EMG records obtained during the experiments with the dynamometer show a level of activity lower than 5% (Fig. 4). However, despite the fact that the examinee is used to the machine, picks of activity appear at the beginning of each flexion, more particularly at low and medium angular velocities. The picks can be attributed to reflex involuntary contractions. As these contractions may bias the estimation the corresponding time-interval are removed from the data-set. In addition the measurements of the dynamometer presents an over-shoot artifact that must be removed too. From the original 210563 samples only 110156 remain (52%).

The EMG records obtained during the constraint-free movements in the motion capture studio (Fig. 5) are lower than 1% and their is a minute variation before the release of the arm and after. In addition, the data obtained with the optical motion capture studio don't need special post-processing. They are used such as described in section III.B.

Interpretation

Condition number of the observation matrix

In both cases, we have $CondW \leq 5$ which shows that W is well-conditioned. Therefore there is a good excitation of the dynamics to estimate.

Relative standard deviation of the estimated parameters

In both cases, parameters are estimated with a low relative standard deviation $\sigma_{\hat{X}j}\%$. Consequently the obtained values are significant.

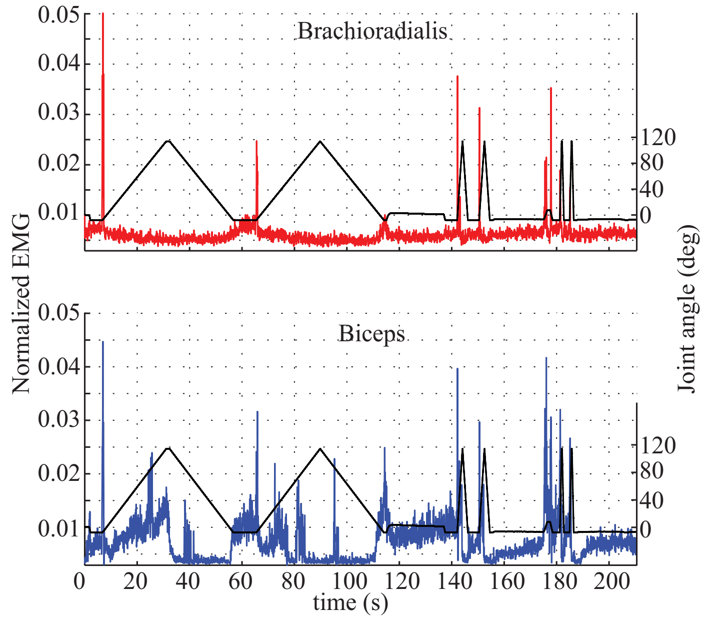

Figure 4: Activity of the Brachioradialis and the Biceps during the isokinetic movements achieve with the dynamometer

Table 2: Results of the estimation for isokinetic movements

parameter	unit	$\hat{X}$	$\sigma_{\hat{X}j}\%$
$cond(W)$		4.1	
Number of samples		110156 of 210563	
stiffness k_s	Nm/rad	1.45	0.42
viscosity h_s	Nms/rad	1.73	0.26
friction f_s	Nm	6.25	0.07
offset o_es	Nm	4.51	0.17

Validations

The direct validation are given in Fig. 6 for the results obtained with the dynamometer and Fig. 7 for the results obtained with the motion capture. The whole data-set for both experimental systems is used (100% of the measured data). They show that the vector of errors ρ (black line) is rather low.

Comparison with literature data

It is not often that the numerical value of the viscoelastic parameters of human joints can be found in the literature and the most cited paper is (Winters and Stark, 1985). The numerical data are given for a 170cm high 70kg male (1% taller and 21% heavier than the examinee in this study): $k_{lit} = 1.5\ Nm/rad$ and $h_{lit} = 0.2\ Nms/rad$. In (Stroeve, 1999) they added that these parameters 'are defined such that they include the intrinsic stiffness and viscosity of (co)-activated muscles'. Comparison with our estimated data show that the stiffness k is in the same range, however the viscosity h is over-estimated in the case of the dynamometer and under estimated in the case of the motion capture system. The under-estimation can be explained by the fact that the viscosity increases when muscles are (co-)activated. Obtained results in (Valle *et al.*, 2006)

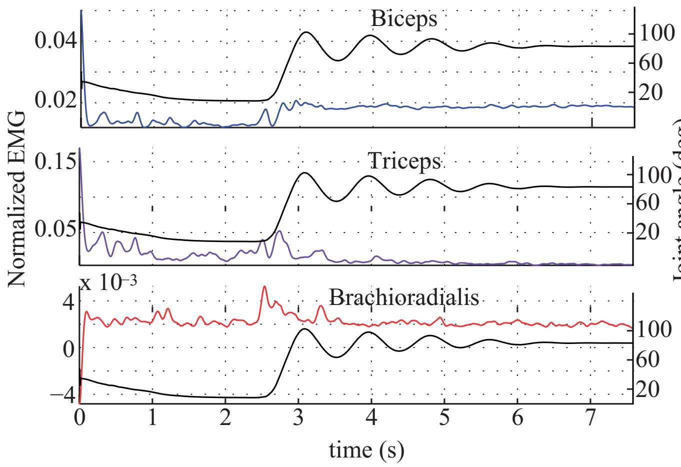

Figure 5: Activity of the Brachioradialis, the Biceps and the Triceps during the constraint-free movements performed in the motion capture studio

Table 3: Results of the estimation for constraint-free movements

parameter	unit	$\hat{X}$	$\sigma_{\hat{X}j}\%$
$cond(W)$		3.6	
Number of samples		902 of 912	
stiffness k_e	Nm/rad	2.24	0.3
viscosity h_e	Nms/rad	0.03	9.4
friction f_e	Nm	-0.01	20
offset o_e	Nm	-0.01	15

for the knee joint viscosity present similar under estimation when they are compared to the results given in (Winters and Stark, 1985), which can mean an over-estimation of this parameter in (Winters and Stark, 1985).

Discussion

Despite the analysis of the statistical indexes given in the previous section when looking at the estimated values there is a clear difference according to the experimental system used. Parameters estimated with the constraint-free movements (Table 3) show that there is a low viscosity and a low friction in the elbow joint, while the elasticity is $k = 2.24 Nm/rad$. With the dynamometer the elasticity (Table 2) is only $k = 1.45 Nm/rad$ while the viscosity and the friction are high and very different than the one that can be found in the literature. With $f = 6.25 Nm$ more than 50% of the passive joint torque is due to friction; moreover it means that flexion/extension movements can be achieved only after overcoming the resistive torque of $6.25 Nm$ which is unrealistic and unacceptable for the human body joints. Consequently the estimated parameters with the dynamometer are not the parameters of the elbow joint only and they also may include the viscoelasticity of the dynamometer itself. This assumption is confirmed by comparing the obtained results with literature data.

The results obtained with the motion capture system rely on the positioning of the optical markers, yet by using anatomical points to set the markers we limit the resulting

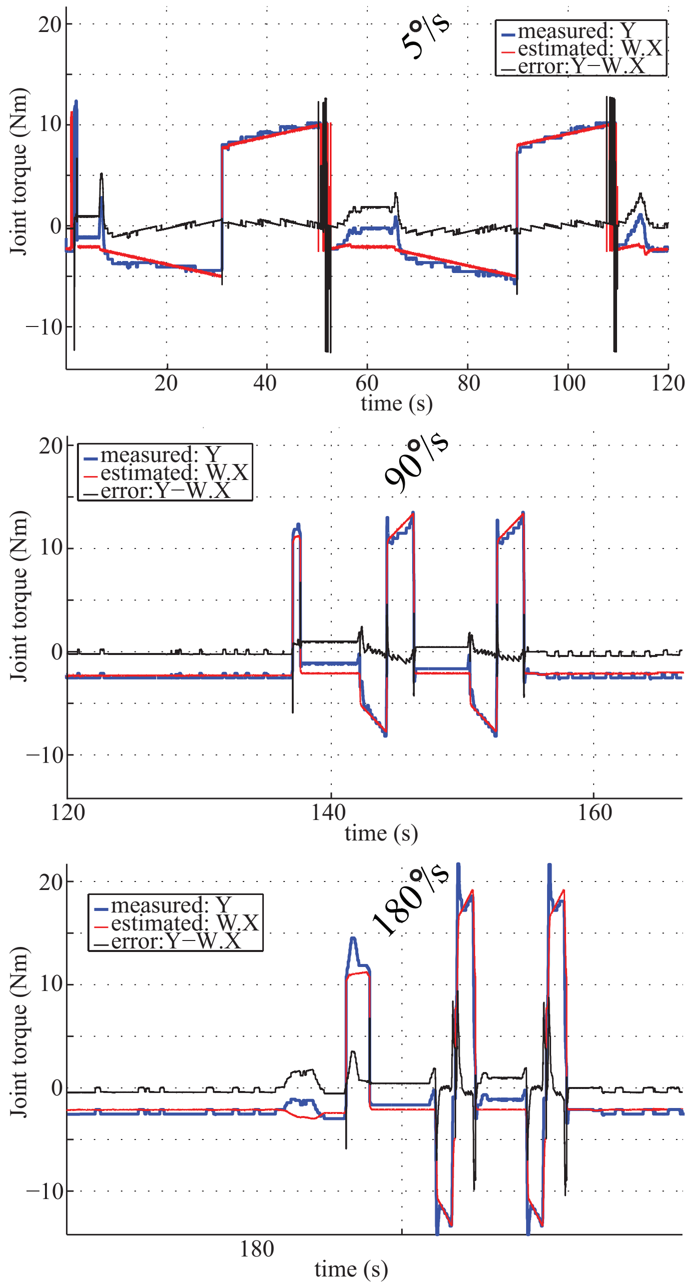

Figure 6: Direct validation for the isokinetic movements achieve with the dynamometer: top, for the first phase of the movement, at $5 deg/s$; center, for the second phase of the movement, at $90 deg/s$; bottom, for the last phase of the movements, at $180 deg/s$

error. The repeatability of the constraint-free movements has been tested and there is a good correlation with less than 3.5% error on the estimated parameters. On the other hand, the obtained results with the dynamometer depend on the calibration, the conservation of the alignment of the dynamometer rotation axis and the joint rotation axis and on the applied force (Prochazka *et al.*, 1997); for all these reasons repeatability with the dynamometer is also an often occurring problem according to (Nuyens *et al.*, 2000).

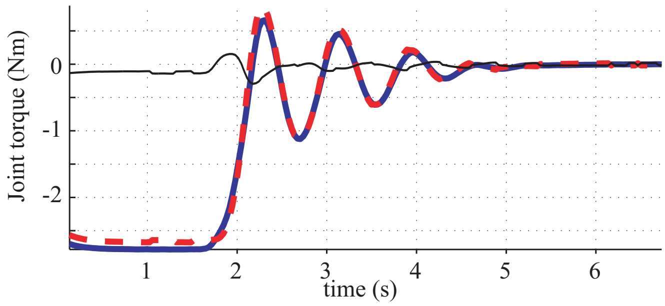

Figure 7: Direct validation for the constraint-free movements recorded in the motion capture studio

Conclusion

The limb joint viscoelastic properties are important data in the fields of biomechanics, rehabilitation, neurology and prothesis design. The most common method used in biomechanics and rehabilitation is based on the use of a dynamometer and the achievement of isokinetic movements. In this paper we have compared the results obtained using this method and a proposed method based on the use of an optical motion capture system with the achievement of constraint-free movements. In both cases the viscoelastic properties of the joint are described using a linear model with constant parameters that can be enhanced to a model where the stiffness parameter is function of the joint angle to take into account nonlinearities at the bounds of the joints. The obtained result have shown that both isokinetic movements and constraint-free movements are reliable for the estimation of the parameters as the condition number of the observation matrix is lower than 5. And in both cases the parameters are estimated with low relative standard deviation which guaranties a good accuracy in the obtained parameters. However the two methods lead to different value of the parameters: more particularly the estimated friction $f = 6.25 Nm$ and viscosity $h = 1.73 Nms/rad$ when using the dynamometer are not realistic as for human body parameters and these estimated parameters may include the dynamometer viscoelastic-properties. Preliminary measurements of the dynamometer dynamics are then required to identify the equipment dynamics. Moreover the data-set obtained with the dynamometer contained artifacts that must be removed, and the EMG signals show some activity at the beginning of flexion movements at low velocities due to stress and reflex contractions. Almost 50% of the data are consequently removed. Therefore rather than the isokinetic movements, the constraint-free movements are more suitable for the identification of the limb joint dynamics. They are painless and easy to perform even by elderly people or people under rehabilitation treatment. The motion capture system can also be used by neurologist during the assessment of muscle rigidity, and it offers the possibility to estimate simultaneously several joints. Finally, the knowledge of the subject specific passive dynamics is a prerequisite to a better understanding of the human motion and particularly in the control of active movements such as in sports.

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