IDENTIFICATION OF THE DYNAMIC PARAMETERS OF A CAR: SIMULATION AND EXPERIMENTAL RESULTS

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Abstract: This paper presents a dynamic modelling of a car using robotic formalism in order to estimate the dynamic parameters of a car and more particularly the dynamic parameters of the chassis and the suspensions. The inverse dynamic model is used to identify the dynamic parameters of the car. They are estimated using a weighted least squares method. Both simulation and experimental results are given for a *406 Peugeot* car.

Key words: parameter identification, automobiles, dynamic modelling, least squares.

1. INTRODUCTION

Car manufacturers need to design their vehicles faster than ever but with a higher range of quality and security to satisfy the customers. Computation and simulation become more frequent particularly during the conception of the vehicle, but they remain poor used for the tuning. This is not because of the lack of tools to simulate the behaviour of the car but only because the car itself is not well known and it is not trivial to correlate computation and real test. This paper presents a method to estimate the dynamic parameters of a car using a simple modelling based on robotic formalism. This method allows the unknown parameters with some trajectories commonly used for the tests and straightforward computations. The modelling of the car and the identification method will be briefly presented. The model has been validated with simulated data obtained with a dynamic model of the vehicle developed by *PSA* which is different from the identification model. Finally the instrumentation required for the tests will be developed, as well as the tests, and practical results are given for a real vehicle: *Peugeot 406*.

2. DYNAMIC MODELLING OF A CAR

2.1. Geometric modelling of a car using the modified Denavit & Hartenberg formalism

The modified *Denavit & Hartenberg* is commonly used in robotics to describe multi-body systems, where bodies are connected with joint that represent one degree of freedom: translation or rotation with respect to the axis of the joint. A passenger car can be modelled (Figure 1) with a 42-bodies tree structure [Venture et al 03] where:

- C_0 is the ground
- C_6 is the chassis. The position and orientation of C_6 w.r.t. the ground define the posture $\xi = [q_1\ q_2\ q_3\ q_4\ q_5\ q_6]^T$ as described by Guillo and Gautier (2000).
- C_{11}, C_{20}, C_{29} and C_{38} are the axles
- C_{14}, C_{23}, C_{32} and C_{41} are the wheels

The dynamic parameters that have to be estimated are the inertial parameters of those bodies: mass, inertia matrix, position of the centre of mass.

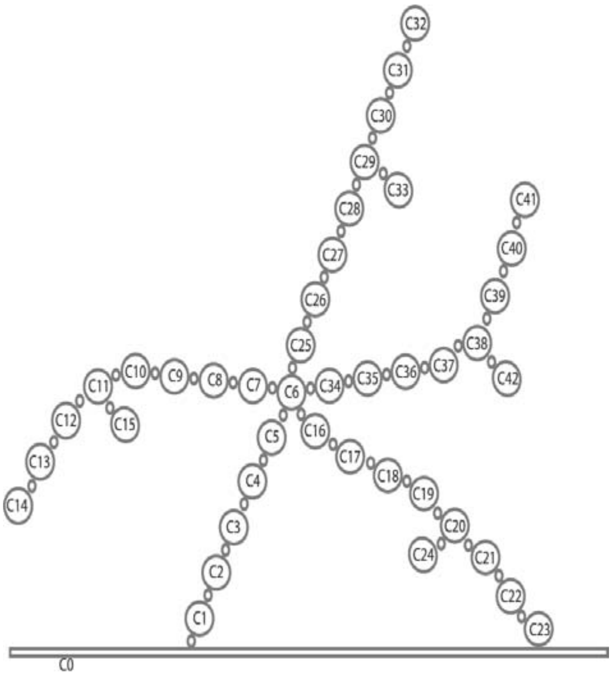

Figure 1 : multi-bodies description of a car

All the other bodies are virtual bodies. They have no mass and no inertia. They are only used to describe degrees of freedom or frames.

Some joints are elastic: they are defined with a stiffness coefficient and also, when it is required a damper coefficient and a friction Coulomb coefficient:

- joints *9*, *18*, *27* and *36* represent the suspensions.
- joints *15*, *24*, *33* and *42* represent the vertical flexibility of the tires.

The associated parameters are estimated too.

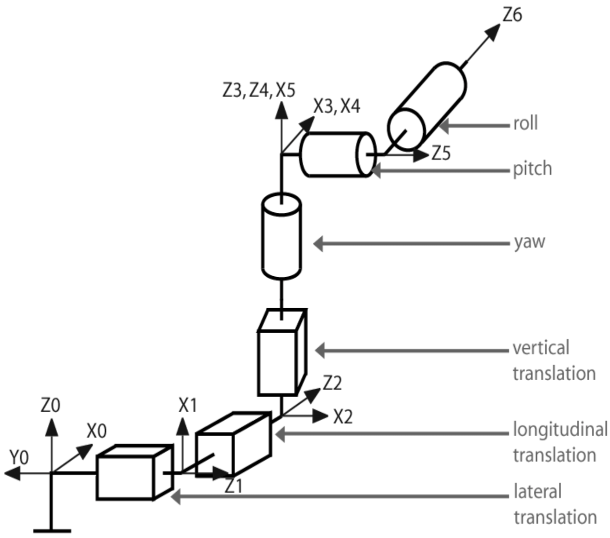

Figure 2 : Modelling of the car posture

The joint variables of the car posture are the following (Figure 2):

- q_1 is the lateral movement of the chassis
- q_2 is the longitudinal movement of the chassis
- q_3 is the vertical movement of the chassis
- q_4 is the yaw angle
- q_5 is the pitch angle
- q_6 is the roll angle

Joint variable of a branch are (Figure 3) :

- q_7, q_{16}, q_{25} and q_{34} are the ½ track width,
- q_8, q_{17}, q_{26} and q_{35} are the ½ wheelbase,
- q_9, q_{18}, q_{27} and q_{36} are the dampers clearances,
- q_{10}, q_{19}, q_{28} and q_{37} are the steering angles,
- q_{11}, q_{20}, q_{29} and q_{38} are the camber angles,
- q_{13}, q_{22}, q_{31} and q_{40} are the kingpin angles
- q_{14}, q_{23}, q_{32} and q_{41} angular position of the wheels
- q_{15}, q_{24}, q_{33} and q_{42} are the tire deflection.

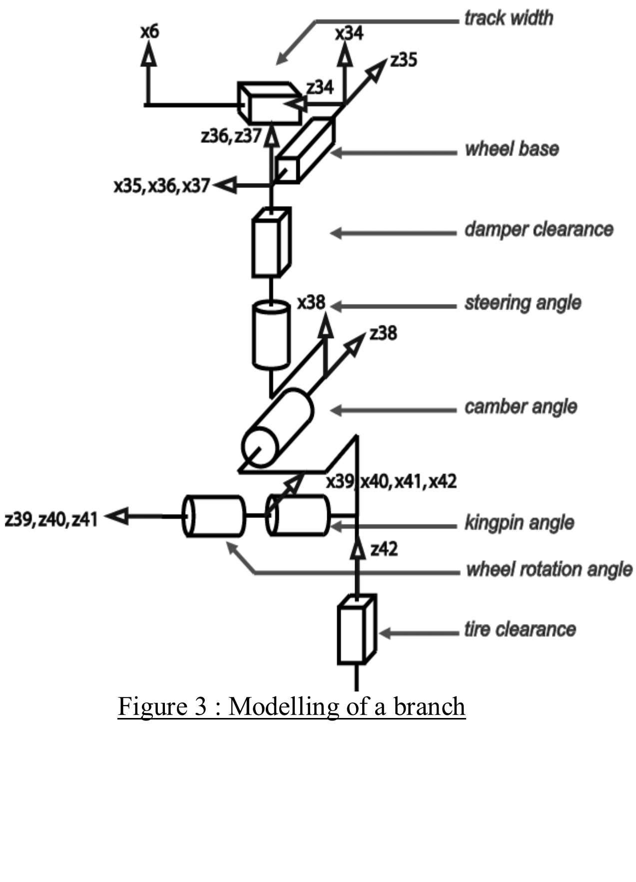

Figure 3 : Modelling of a branch

This description allows calculating the geometric, kinematic and dynamic models systematically using well known algorithms that are developed for robotics applications (Khalil and Dombre 2002), or even automatically with the help of *SYMORO*$^{+}$, software of symbolic calculations developed by Khalil and Creusot (1997) in the robotics team of the *IRCCyN.*

3. DYNAMIC MODELS

The inverse dynamic model gives the joint torques as a function of the joint positions, velocities and accelerations.
Let the Inverse Dynamic Model (*IDM*) be written as:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{L} + \mathbf{Q}^{c} \quad (1)$$

- $\mathbf{M}(\mathbf{q})$ the mass matrix of the system Σ
- $\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}})$ the vector of centrifugal, *Coriolis* and gravity terms.
- **L** the vector of the internal forces between the vehicle bodies: motor torque, friction, lumped elasticity.
- $\mathbf{Q}^{c}$ the vector of the contact forces projected on the joint axes.

3.1. Identification model

The dynamic model (1) can be expressed as a linear relation w.r.t. the identifiable parameters (inertial parameters of the real bodies and parameters of the elastic joints). It can be written as:

$$\mathbf{y} = \mathbf{D}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}).\mathbf{X} \quad (2)$$

3.2. Identification method

The dynamic parameters are estimated by solving an over-determined linear system with the weighted least squares techniques (*W.L.S.*). The system is obtained by sampling the identification dynamic model (2) along a trajectory.
The whole system is written as follows:

$$\mathbf{Y} = \mathbf{W}.\mathbf{X} + \boldsymbol{\rho} \quad (3)$$

- **W** is the observation matrix (n_{exp} x n_b)
- **Y** the (n_{exp} x *1*) vector of joint and reaction forces
- ρ the (n_{exp} x *1*) vector of errors
- **X** the vector of the dynamic parameters to identify (n_b x *1*)

Under the hypothesis of a zero mean and gaussian error, this method allows to estimate a confidence interval for each parameter.

4. VALIDATION OF THE MODEL WITH A DYNAMIC VEHICLE SIMULATION SOFTWARE

4.1. Aim and interest

Manufacturers have already developed car simulators. They are based on the direct dynamic model, which is non linear with respect to the joint variables and where the dynamic parameters must be given. They are used to simulate the car behaviour as a function of the steering angle and longitudinal speed. One of the most complete model is used in the software *ARHMM* (Bodson 2002), and this is the one that is used to validate the approach presented in section 2.
The aim of the validation is to estimate, with the presented method, the dynamic parameters of the simulation model and to compare the results obtained with the known parameters of the simulation model.

4.2. Trajectories

Two trajectories are chosen, among many, for the validation, in order to excite all the parameters: a sinus steering for the yaw and roll behaviour and a straight line test with sudden deceleration without braking for the pitch behaviour.

4.3. Results and validation

The last columns of the tables 1 and 2 give the relative standard deviation: *sxr-%*. If *sxr-%* is too high *(sxr-%>1.5),* the parameter is not well identified.
Inertia matrix is given in the center of mass frame (**x** longitudinal, **y** lateral and **z** vertical), besides the position of the centre of mass is given in the architectural frame R_Q (**x** longitudinal from front to rear, **y** lateral and **z** vertical) and in millimetres.
Relative standard errors given for each parameter are a good indicator for the interpretation of the results but it is important to validate the results with respect to the measurements.
A validation method consists in estimating the vector of errors ρ which is performed by comparing **Y** to the reconstructed vector **W.X** according to (3).

Table 1 : Results of the identification, simulation : parameters of the hole vehicle

parameter	units	à priori	**estimated**	*sxr %*
XX_6	*Kg.m²*	***630***	**644.66**	0.96
YY_6	*Kg.m²*	***2250***	**2329.26**	0.14
ZZ_6	*Kg.m²*	***2734***	**2840.83**	0.52
XY_6	*Kg.m²*	***13***	**1.01**	5.91
XZ_6	*Kg.m²*	***20***	**30.54**	2.01
YZ_6	*Kg.m²*	***-76***	**0.00**	4.18
$X_{G/RQ}$	*mm*	***1112.5***	**1111.06**	0.002
$Y_{G/RQ}$	*mm*	***0***	**-5.16**	38.05
$Z_{G/RQ}$	*mm*	***259.7***	**261.43**	2.00
M_6	*Kg*	***1668.5***	**1668.55**	0.07

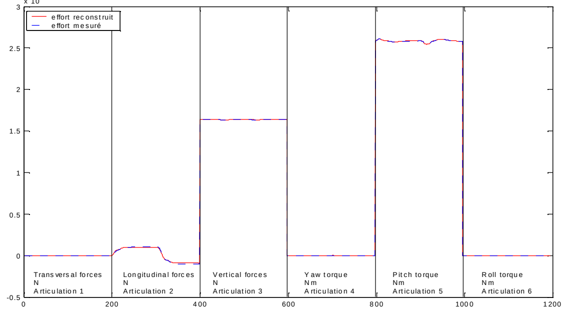

Figure 4 : Validation with the straight line, chassis

Table 2 : Results of the identification, simulation : parameters of the suspensions

parameter	units	à priori	estimated	sxr %
ks_9	*N/m*	***22000***	**22295.78**	0.02
hs_9	*N/m/s*	***3800***	**3823.86**	0.07
off_9	*N*		**9639.46**	0.01
ks_{18}	*N/m*	***22000***	**22193.35**	0.06
hs_{18}	*N/m/s*	***3800***	**3843.34**	0.02
off_{18}	*N*		**9474.54**	0.01
ks_{27}	*N/m*	***20000***	**20683.35**	0.06
hs_{27}	*N/m/s*	***3200***	**3198.86**	0.02
off_{27}	*N*		**10421.41**	0.01
ks_{36}	*N/m*	***20000***	**20604.04**	0.02
hs_{36}	*N/m/s*	***3200***	**3188.27**	0.07
off_{36}	*N*		**10512**	0.01

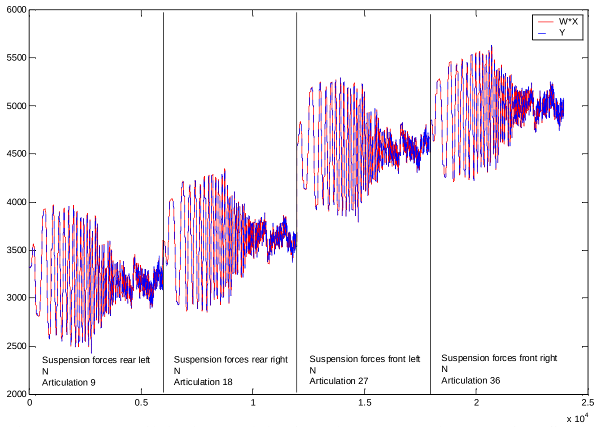

Figure 5 : Validation with the sinus steering 90 km/h, suspensions parameters

Table 3 : Results of the identification, simulation : parameters of the wheels

parameter	units	à priori	estimated	sxr %
ZZ_{14}	*Kg/m²*	***1***	**1.02**	1.50
ZZ_{23}	*Kg/m²*	***1***	**1.02**	1.40
ZZ_{32}	*Kg/m²*	***1***	**-51.72**	5.42
ZZ_{41}	*Kg/m²*	***1***	**-51.60**	4.78

Table 4 : Results of the identification, simulation : parameters of the tires

parameter	units	à priori	estimated	sxr %
ks_{15}	*N/m*	***200000***	**203483.93**	0.07
ks_{24}	*N/m*	***200000***	**196251.29**	0.08
ks_{33}	*N/m*	***200000***	**206040.0**	0.12
ks_{42}	*N/m*	***200000***	**194402.9**	0.13

4.4. Interpretation

Simulation results show that the *DHM* model of the car is quite equivalent to a dynamic vehicle model used in the simulation software *ARHMM*. The identification model obtained gives a good representation of the car behaviour and allows to estimate the dynamic parameters of the chassis and the wheels and also the parameters of the suspensions and the vertical stiffness of the tires. The identification model can now be used on a real car. Simulations have also helped in defining the trajectories that will be used for the identification, in order to have a good excitation of the parameters.

5. EXPERIMENTAL RESULTS

5.1. Measurement and post-treatment

Data acquisitions are performed on a real car equipped with various sensors for constructing the joint variables and the contact forces and moments. Those sensors are four dynamometric *Igel* wheels, a *Sagem* inertial unit, laser sensors for vertical position, *Zimmer* video cameras for the measure of the steering angles and the camber angles, and a *Correvi*t speed sensor. Other parameters are computed by tables using the damper clearances that are obtained with test bench. Trajectories are chosen for their dynamic properties among standard tests in order to have no additional cost.(Venture et al 2003)

5.2. Measurement and filtering

All the measurements (position, velocity or acceleration) are filtered in order to estimate the position, the velocity and the acceleration for each joint. The filter chosen is a pass band filtering. This operation is carried out using the function *filtfilt* of *Matlab* with an order between 5 to 8 and a cut-off frequency chosen according to the dynamic of the system and the sensor. Usually it is about 3 to 7 hertz for the car. (Gautier, 1997).

The derivatives are estimated without phase shift using a central difference algorithm, and the integration is estimated with a trapeze method.

6. RESULTS AND INTERPRETATION

6.1. Results

The estimated values and the relative standard deviation are shown in tables 2, 3, and 4. They are obtained using sinus steering at *90 Km/h*, and spiral steering tests. The sample time is *0.016* second. A test last from 5 seconds to one minute. But most of the tests used here last about one minute.

Table 5: Results of the identification, experimental : parameters of the hole vehicle

parameter	units	à priori	estimated	sxr %
XX_G	*Kg.m²*	***2734***	**2783.96**	0.36
ZZ_G	*Kg.m²*	***630***	**760.31**	1.35
XZ_G	*Kg.m²*	***20***	**45**	1.16
$X_{G/RQ}$	*mm*	***1112.5***	**1131.06**	0.06
$Y_{G/RQ}$	*mm*	***0***	**-5.16**	0.80
$Z_{G/RQ}$	*mm*	***259.7***	**246.69**	1.14
M_6	*Kg*	***1668.5***	**1668.53**	0.07

Table 6: Results of the identification, experimental : parameters of the suspensions

parameter	units	à priori	estimated	sxr %
fs9	***N***		**19.80**	4.94

off9	*N*		**11376.21**	0.91
ks9	*N/m*	*22000*	**27059.51**	1.32
h9	*N/m/s*	*4200*	**4526.08**	1.24
fs18	*N*		**44.91**	1.93
off18	*N*		**9992.02**	0.97
ks18	*N/m*	*22000*	**23317.02**	1.44
h18	*N/m/s*	*4200*	**4274.80**	1.07
karr	*N/m*	*19185*	**24785.43**	0.70
fs27	*N*		**32.94**	3.04
off27	*N*		**12890.90**	0.75
ks27	*N/m*	*20000*	**27129.51**	1.18
h27	*N/m/s*	*3200*	**3306.20**	1.36
fs36	*N*		**50.35**	1.92
off36	*N*		**13103.00**	0.77
ks36	*N/m*	*20000*	**27084.50**	1.24
h36	*N/m/s*	*3200*	**3408.44**	1.21
karf	*N/m*	*19780*	**18394.90**	0.89

Table 7: Results of the identification, experimental : parameters of the tires

parameter	units	à priori	**estimated**	*sxr %*
kt15	*N/m*	*200000*	**216502.50**	0.32
kt24	*N/m*	*200000*	**217816.21**	0.31
kt33	*N/m*	*200000*	**232581.41**	0.37
kt42	*N/m*	*200000*	**230549.08**	0.37

6.2. Interpretation and validation

Figures 6, 7 and 8 give the results of a direct validation with a sinus steering – for the chassis and the suspensions – or spirals – for the vertical stiffness of the tires. Figures 9 and 10 give the results of a cross validation, where the trajectory used is not the same as the one for the estimation: the test used for the validation is a braking in straight line.

Pitch inertia cannot be identified with a sinus steering. It is numerically eliminated, and this can be seen on Figure 9 with the cross validation using straight line braking, on the 5th joint, there is bad correlation between **Y** and **W.X**.

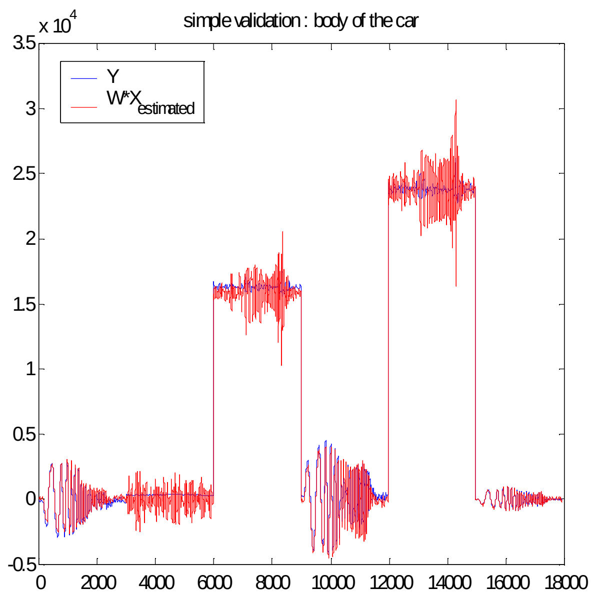

Figure 6: direct validation for the chassis

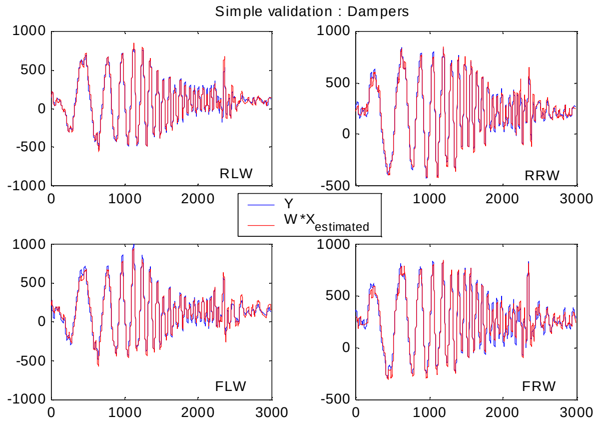

Figure 7: direct validation for the suspension

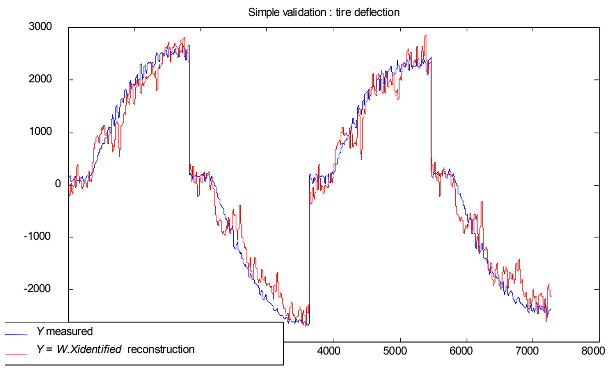

Figure 8 : direct validation for the vertical stiffness of the tires

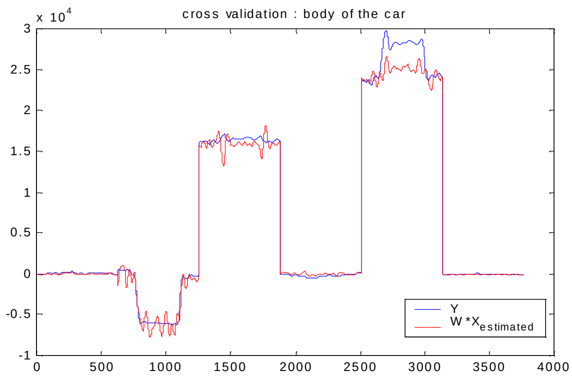

Figure 9 : cross validation for the chassis

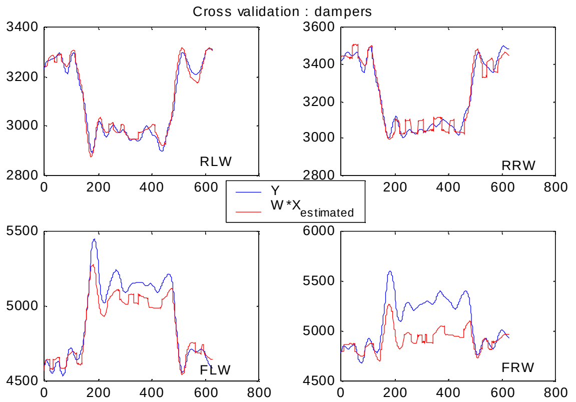

Figure 10 : cross validation for the suspension

7. CONCLUSION

This paper has presented an original method, based on robotic formalism, of modelling a car in order to estimate its dynamic parameters. Contrary to methods commonly used for this task, this method presents many advantages such as:

- A good representation of the dynamic behaviour of the car that has been validated with simulation software,
- A low cost of computations by using a model which is linear with respect to the dynamic parameters and a least squares method
- The method needs no initialisation for the parameters, thus no need for a priori value of the parameters.

The validation of the results is performed with different tools: first the analyse of the relative standard deviation given for each parameter, then the estimation of the vector of errors by comparison of **Y** and **W.X** for different trajectories.

The weakness of the actual implementation of the method is the obligation of using dynamometric wheels to measure the ground contact efforts applied on the vehicle. An observer can be built and is the object of further work.

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